Bidding Behavior in Multi–Unit Auctions — An Experimental Investigation*

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Abstract

We present laboratory experiments of five different multi–unit auction mechanisms. Two units of a homogeneous object were auctioned off among two bidders with flat demand for two units. We test whether expected demand reduction occurs in open and sealed–bid uniform–price auctions. We also test revenue equivalence for these auctions as well as for the Ausubel, the Vickrey and the discriminatory sealed–bid auction. Furthermore, we compare the five mechanisms with respect to the efficient allocation of the units.

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1 Introduction

Auctions play a more and more important role in everyday business life. An increasing number of important markets are organized as auctions and auction design is probably one of the most successful applications of economic theory to practice. Among the prominent examples are the spectrum auctions in the US and in Europe, treasury auctions in the United States, the design of electricity spot markets, oil and timber sales as well as the massive use of procurement auctions, both by the public sector and by private firms. The spectrum auctions across the world have demonstrated that carefully designed auctions raise high revenues and often succeed in implementing efficient allocations. In Germany and the UK, for example, the 3rd generation spectrum auctions in 2000 raised 50.8 and 37.5 billion Euros, respectively.[1]

A common feature of most real world auctions is that multiple units of an object are sold, either simultaneously, or in quick succession. Unfortunately, theoretical analyses of multi–unit auctions are generally plagued by multiplicity of equilibria which impede revenue and efficiency comparisons of different auction rules. Moreover, in particular if multiple units are sold, the incentives to enter and to collude in different auction formats are crucial for their performance. As Klemperer (2002b) points out, these issues are often actually more important than those auction theorists tend to be concerned with.[2] In the experimental literature single–unit auctions have been thoroughly investigated, but there are only few contributions that focus on multiple–unit auctions. Hence, in order to provide a basis for good market design, further theoretical and experimental studies are needed that shed light on revenue and efficiency properties of multi–unit auction rules.

Our paper contributes to closing this gap. We experimentally investigate bidding behavior in five different multi–unit auction formats and provide efficiency and revenue com-

[1]See Grimm et al. (2004).

[2]For example, he argues that because of problems related to entry and collusion, several European spectrum auctions did not perform as expected, leaving the auctioneer with revenues close to the reserve prices. In particular in Austria and Switzerland, bidders coordinated on an allocation without much bidding activity. Those auctions generated only 100 and 20 Euros per capita, respectively, as compared to 615 (650) in Germany (the UK). See Klemperer (2002a, 2004).

parisons in a unified experimental framework. We analyze the discriminatory auction (DA), the uniform–price sealed–bid auction (UPS), the uniform–price open auction (UPO), the Vickrey Auction (VA), and the Ausubel Auction (AA). Our experiment consists of a series of two–unit, two–bidder auctions. Bidders have flat demand for two units. Although stylized, the framework provides a stress test for several central hypotheses in multiple–unit auction design.

First, our experiments shed light on the issue of demand reduction in uniform–price multi–unit auctions. These auctions generally have multiple equilibria, where bidders reduce demand on all but the first unit they bid on. Demand reduction has first been shown as an equilibrium phenomenon by Wilson (1979) and Back and Zender (1993) in the context of share auctions.[3] Noussair (1995), Katzman (1995), Engelbrecht–Wiggans and Kahn (1998), Ausubel and Cramton (2002) and Grimm et al. (2003) analyze auctions where bidders have demand for multiple non–divisible units and give examples of equilibria that involve demand reduction. Brusco and Lopomo (2002), and Grimm et al. (2003) show that open ascending auctions have collusive equilibria that imply strong inefficiencies and extremely low prices, i.e. bidders may succeed to coordinate on a low price equilibrium. In our experiment we demonstrate that several bidders indeed follow this pattern.

Our experiments also contribute to the ongoing debate whether open ascending or sealed–bid auctions are preferable. Klemperer (2002b) argues that collusive outcomes are more difficult to achieve in sealed–bid auctions.[4] This would imply that sealed–bid auctions yield a higher revenue than open auctions, which is clearly confirmed by our experimental data. From a revenue perspective it seems to be much more important whether the auction is open or sealed–bid than which pricing rule it has.[5] Our observations thereby confirm the conclusions Klemperer (2002b) draws from auctions in the field that the incentives to collude are more relevant for practical auction design than some of the issues at the focus of traditional auction theory.

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[3]See also Menezes (1996) and Wang and Zender (2002).

[4]See also Cramton and Schwartz (2000, 2002) on this issue.

[5]Overbidding is substantial in UPS and DA which yields above equilibrium revenues on average. Initial attempts to collude in AA lower revenue in the first periods leading to average revenues below the equilibrium prediction.

However, in many markets open ascending auctions are for several reasons considered the most appropriate format. They are arguably more transparent to bidders with respect to how their bids determine both the number of units they obtain and the prices they pay.[6] Furthermore, open ascending auctions have very attractive informational properties.[7] In the light of (theoretically) low efficiency and revenue predictions of uniform–price auctions (and to provide an open analogue to the multiple–unit Vickrey auction), Ausubel (2004) has proposed an incentive compatible open multi–unit auction that is superior to the uniform–price auction in terms of efficiency. Our experimental evidence on the Ausubel auction is well in line with this. Our data show that tacit collusion obtains if it is supported by an equilibrium outcome (i.e. in the uniform–price auction), whereas initial attempts to collude are not successful in the Ausubel auction where the unique equilibrium prediction is truthful bidding. We also find less overbidding in AA than in VA, supporting the view that the open auction provides better information to bidders and hence leads to fewer deviations from equilibrium and higher efficiency.

Furthermore, our results contribute to long lasting debates in many markets (e.g. treasury bills and electricity markets) whether uniform–price or pay–as–bid (i.e. discriminatory) auctions should be used.[8] In our experiment we find no significant differences between sealed–bid uniform and discriminatory auctions with respect to revenue and efficiency (the uniform–price open auction performs significantly worse in both respects due to the higher degree of demand reduction). However, especially for the discriminatory auction we observe behavioral patterns that systematically differ from key features of the equilibrium bid function. In particular, we provide a test of a fundamental equilibrium property of the discriminatory auction. A robust result for pay–as–bid auctions is that a bidder's submitted

[6] See Milgrom (2000), or Klemperer (2003) on various arguments why an open ascending auction is considered the most appropriate format to allocate radio spectrum all over the world, although the existence of inefficient low price equilibria presents a significant problem (see Klemperer (2002a) on the latter issue).

[7] If the bidders' valuations are affiliated, an open ascending auction reduces the amount of private information held by the bidders and thereby their informational rents.

[8] See, e.g. Friedman (1960) for the Treasury auctions, or Kahn et al. (2001) for the Californian electricity market.

demand curve exhibits flat parts.[9] In our experimental setting it is completely flat, i.e. a bidder places two identical bids. Our experimental evidence, however, completely contradicts this theoretical prediction. Indeed, bid spreading is most pronounced in the discriminatory auction among all five formats we study. We argue that this might be caused by a myopic joy of winning. Given the prominence of multi–unit discriminatory auctions in applications, such systematic deviations from equilibrium appear to be of substantial interest.

To summarize, our experiments provide several clear conclusions that are helpful for the design of multi–unit auctions. We confirm, in a controlled laboratory setting, crucial theoretical results on demand reduction in uniform–price auctions as well as observations from the field that open uniform–price auctions are more susceptible to collusion. We also confirm desirable properties of the Ausubel auction, but at the same time show that this open format leads to (unsuccessful) attempts to collude. Hence we highlight the attractiveness of this auction mechanism (as well as its potential risks). Furthermore, we find unexpected behavior in pay–as–bid auctions, which can have substantial negative effects on efficiency and should give rise to caution in using this auction format in the field on the basis of exclusively theoretical arguments.

Closely related experiments were run by Alsemgeest et al. (1998), Kagel and Levin (2001), List and Lucking–Reiley (2000), Porter and Vragov (2006), Manelli et al. (2006), and Kwasnica and Sherstyuk (2007). Our experiments are, however, the first to compare all these five standard auction formats in the same framework. Alsemgeest et al. (1998) compare UPO and a version of UPS (with a different pricing rule). They find that revenue is higher in the sealed–bid auction and that bidders reduce demand in UPO. Kagel and Levin (2001) compare uniform–price sealed–bid and open auctions and the Ausubel auction and find systematic demand reduction in the uniform–price auctions. Their subjects also have flat demand for two units but bid against robot bidders with unit demand. List and Lucking–Reiley (2000) conduct field experiments, comparing the uniform–price sealed–bid and the Vickrey sealed–bid auction by selling sportscards in two–unit, two–person auctions. They also find demand reduction in uniform–price auctions, compared to Vickrey auctions. They cannot, however, control for the bidders' valuations. Their experiment is

[9]See, e.g., Lebrun and Tremblay (2003).

replicated in the laboratory by Porter and Vragov (2006), who find substantial deviations from demand revelation in both, UPS and VA, but also more demand reduction in UPS. Manelli et al. (2006) compare the Ausubel and the Vickrey auction and also find that revenues are higher in the sealed–bid auction whereas efficiency of both mechanism is the same. Kwasnica and Sherstyuk (2007) study tacit collusion in ascending auctions and find that payoff–superior (collusive) equilibria are more likely to be played if complementarities are not too strong and the number of bidders is small.

The paper is structured as follows. Section 2 presents the equilibria of our auction games and the implied hypotheses. The experimental design is presented in section 3, followed by the experimental results in section 4, a discussion in section 5, and the conclusions.

2 Theoretical Background and Hypotheses

2.1 Equilibria of the Five Auction Formats

We investigate bidding behavior in independent private value auctions with two bidders and two indivisible identical objects for sale. Each bidder demands at most two units. A bidder i places the same value v_i on each unit. The bidders' valuations are drawn independently from the same uniform distribution on the interval $[0, V]$.

We consider five different auction formats. In the three sealed–bid auctions the bidders simultaneously submit sealed bids for each of the units demanded and prices and allocations are determined according to the auction rules. The two open auctions start out with a price of zero and active bids on all units demanded. The price is increased and units are traded according to the rules of the mechanism as bidders drop out. In all auctions the two highest bids win a unit each.

Our design choices are motivated by the following considerations. First, the rules for the auction (in particular for the Vickrey auction and the Ausubel auction) are rather complex, and more so the larger the number of objects for sale. Because we wanted to minimize subjects' confusion to influence the results, we chose to auction off only two units. Also the choice of flat valuations is partly motivated by the desire to keep the design as simple as possible. Flat valuations appear easiest to understand. Second, the incentive to reduce demand is arguably the more transparent the higher the number of units. Our design with only two units intentionally tests the minimal case where demand reduction could occur. If it occurs in this setting, it is likely to occur also if a higher number of units is sold (except for the caveat that more units might introduce more subject confusion). Third, flat valuations also allow for clear predictions in the auction formats where we expect no demand reduction. Specifically, both bids are predicted to be equal in all but the uniform price auctions.

Uniform–Price Sealed–Bid Auction [UPS] and Uniform–Price Open Auction [UPO]

Rules. In the uniform–price auctions the price for all units equals the highest rejected bid. In our experiment, this is the third highest bid. In UPS, each bidder places two bids and the units are allocated to the two highest bids (or randomly in case of a tie). UPO starts out with a price of zero, with the price increasing continuously thereafter. Bidders start out actively bidding on two units each and may choose the price(s) where they drop out on one unit, or on both. Dropping out is irrevocable so that a bidder can no longer bid on a unit he has dropped out on. As soon as the number of active bids equals the number of units available, both items are sold to the bidder(s) holding the active bids at the price at which the last bidder dropped out. Thus, the price is determined either by a second dropout of a bidder on one unit or by a bidder's simultaneous dropout on both units.

Equilibrium. In both uniform–price formats any strategy that is not weakly dominated implies to bid one's valuation v_i on the first unit[10] (i.e. the higher bid always equals the true valuation). The argument is identical to that for single–unit Vickrey auctions (see, e.g. Vickrey, 1961).[11]

[10] "First unit" ("second unit") always refers to the unit on which the bidder places the higher (lower) bid.

[11] Note that UPO ends for sure when the price reaches a bidder's first–unit bid. Thus, deviating from bidding one's valuation on the first unit cannot trigger any beneficial behavior of the other bidder. It follows — as for the single–unit Vickrey auction — that a strategy prescribing any other bid than v_i on the first unit is weakly dominated by one that is identical except for bidding v_i on the first unit.

Lowering the bid on the second unit, however, presents a trade–off. A lower bid on the second unit lowers the chance of winning two units but, at the same time, may reduce the price paid for the first unit. As it turns out, the uniform–price auctions have multiple equilibria. All equilibria that do not involve truthful bidding on the first unit are weakly dominated. Among those equilibria that involve truthful bidding on the first unit the following are the extreme cases: Truthful revelation on both units,

$$b_1(v_i) = b_2(v_i) = v_i, \tag{1}$$

(where b_1 denotes the first–unit bid and b_2 the second–unit bid) and full demand reduction on the second unit such that the bid on the second unit is zero,

$$\begin{aligned} b_1(v_i) &= v_i, \\ b_2(v_i) &= 0. \end{aligned} \tag{2}$$

In the following we will refer to these equilibria as the truth–telling (TT) and the demand reduction (DR) equilibrium, respectively.[12] The DR–equilibrium payoff dominates all other equilibria in undominated strategies.

Discriminatory Auction [DA]

Rules. In the discriminatory auction, the two highest bids win a unit each and the respective prices equal these bids.

Equilibrium. An important observation in order to derive the optimal strategy is that with flat demand a bidder places the same bid on both units.[13]

Thus, the equilibrium bid function on each unit solves $\max_b F(\sigma(b))[v_i - b]$, where $\sigma(b)$ is the inverse of the equilibrium strategy $b_i(v)$, $i = 1, 2$. In the case of uniformly distributed valuations on $[0, V]$ and two bidders the equilibrium bid functions are simply

$$b_1(v_i) = b_2(v_i) = \frac{1}{2}v_i. \tag{3}$$

[12] In the working paper version, Engelmann and Grimm (2004), we characterize all equilibria of this game that are not in weakly dominated strategies. These imply partial demand reduction for some values and truth–telling for others. We argue that these equilibria are not plausible, since it is unclear how subjects could coordinate on any of them. Furthermore, the TT– and DR–equilibria are clearly focal.

[13] See Lebrun and Tremblay (2003) for a formal proof of this fact for more general demand functions.

Vickrey Auction [VA]

Rules. In the multi–unit generalization of the Vickrey auction the total price a bidder pays for the units he obtains equals the sum of the bids (other than his own) that are displaced by his successful bids. In our framework this means that, if one bidder places the two highest bids, he pays the two bids of the other bidder. If each bidder places one of the two highest bids, each pays the lower bid of the other bidder because his higher bid displaces the lower bid of the other bidder.

Equilibrium. In VA it is hence a weakly dominant strategy to bid truthfully on both units. See Vickrey (1961) for a proof for the general case. We have

$$b_1(v_i) = b_2(v_i) = v_i. \tag{4}$$

Ausubel Auction [AA]

Rules. The Ausubel (or dynamic Vickrey) auction (Ausubel, 2004) is an open mechanism that implements the same outcome as the multi–unit Vickrey auction in a way that has a great potential for transparency to bidders. The auction starts out at a price of zero which is then increased continuously. In the general case, at any price it is checked for each bidder whether the aggregate demand of the other bidders is smaller than the available number of units. If this is the case, the bidder receives the available units at the current price.

In our case, the price is raised until one bidder (say, bidder i) drops out on one unit. At this point bidder j gets one unit for sure (in other words: he has "clinched" one unit). This unit is traded immediately and bidder j pays the price at which he has clinched it. Then the auction continues at this price for the remaining item that is still unsold. From that point on the two bidders are involved in a single–object English clock auction.

Equilibrium. Under these rules the bidders have an incentive for full demand revelation on both units since the price paid for the first unit does not affect the price paid for the other unit. Thus,

$$b_1(v_i) = b_2(v_i) = v_i. \tag{5}$$

This equilibrium is obtained by iterated elimination of weakly dominated strategies. If one bidder has already dropped out it is weakly dominated to drop out at a price other than v_i, since the dropout price only determines the price for the remaining unit. One can only lose by staying in above v_i and can miss a possible gain by dropping out before v_i is reached. Eliminating these strategies then implies that the price of the first dropout does not influence the result of the subsequent bidding process. Hence it is also weakly dominated to drop out first at a price other than v_i since this dropout price only determines the price for this unit. To make not dropping out at a price lower than v_i optimal, however, requires knowing that the other bidder will not play a dominated strategy (e.g. will not drop out immediately after). Hence the equilibrium is not in weakly dominant strategies, but the game is only dominant solvable. The solution concept is thus weaker than in VA. In contrast the mechanism appears to be more transparent, which might compensate, in terms of efficiency, for the weaker equilibrium concept (see also Kagel et al., 2001).

2.2 Hypotheses Derived from the Theory

The theoretical analysis gives us several hypotheses to test.

(H1) First–unit bids in UPO, UPS, AA, and VA should equal the valuation (see section 4.1 for the results).

(H2) Based on the payoff–dominance of the DR–equilibrium over the remaining equilibria of UPO and UPS, we expect to observe demand reduction on the second unit in the uniform price auctions, at least in some of the pairs. Furthermore, we hypothesize that the DR–equilibrium is chosen more frequently in UPO than in UPS, since it is the only equilibrium of UPO that satisfies certain refinements.[14] In addition, in UPO one bidder can initiate it by dropping out on one unit immediately (see section 4.2 for the results).

[14] We discuss this in more detail in the working paper version, Engelmann and Grimm (2004). Specifically, the DR–equilibrium is the unique Perfect Bayesian equilibrium equilibrium of UPO if the beliefs strictly follow Bayes' rule also off the equilibrium path (that is, if a bidder observes a dropout he infers only that the opponent's valuation is higher than the dropout price and updates the initial distribution accordingly).

(H3) In AA, VA, and DA, the bids on both units should be equal (see section 4.3 for the results).

(H4) In equilibrium, all units should be allocated efficiently in VA, AA, and DA. In contrast, only half of the units should be allocated efficiently if the DR–equilibrium is played in UPO and UPS, or more generally, not all units should be allocated efficiently if (H2) holds (see section 4.4 for the results).

(H5) Revenues are expected to be significantly lower in the uniform–price auctions than in the other three auctions (if (H2) holds). Revenues in AA, VA, and DA are theoretically equivalent in our setting (see section 4.5 for the results).[15]

(H6) The bidders' expected payoffs are equal in AA, VA, and DA. If (H2) holds, they are higher in the uniform price auctions and higher in UPO than in UPS (see section 4.6 for the results).

(H7) Bidders are expected to select the DR–equilibrium more often in UPS if they have played UPO before, i.e. they should manage to transfer the DR they may learn in UPO at least partly to UPS (see section 4.7 for the results).

3 Details of the Experimental Implementation

In each auction the bidders' private valuations for both units were drawn independently from the uniform distribution on $[0, 100]$.[16] The bidders were undergraduate students from Humboldt University Berlin, the University of Zürich, and the ETH Zürich. Pairs of bidders were randomly formed. In DA, VA and AA each pair played ten auctions under the same rules. In the uniform–price auctions, in treatment UPOS each pair first played ten open

[15]In DA, the price for each unit is $\frac{1}{2}\max\{v_i, v_j\}$ and $E[\max\{v_i, v_j\}] = \frac{2}{3}V$. In AA and VA the price is $\min\{v_i, v_j\}$ and $E[\min\{v_i, v_j\}] = \frac{1}{3}V$, so that the expected revenue is $\frac{2}{3}V$ in both cases. In contrast, the expected revenue in the uniform–price auctions is between 0 (if the DR–equilibrium is played) and $\frac{2}{3}V$ (if TT is played).

[16]Valuations were in fact drawn from the set of integers in [0,100] and also bids were restricted to integers. Given this fine grid, the equilibrium predictions are not affected.

auctions and then ten sealed–bid, in treatment UPSO vice versa.[17] Apart from this, in each session only one type of auction was conducted. For each treatment we had ten pairs, except for treatment DA, where we had nine.

The rationale for the fixed matching we employed was twofold. First, there are obvious practical considerations, namely generating a relatively large number of independent observations for each of our five treatments with limited financial and subject pool resources. Second, we believe this to be a tougher test than random matching for the expected theoretical efficiency superiority of AA, VA, and DA. We discuss this point in detail in section 4.4. In addition, for many applications, fixed matching appears to be the more realistic model and it is hence interesting to study the susceptibility of the different auction formats to collusion under repeated interaction.

Subjects were placed at isolated computer terminals, so that they could not determine whom they formed a pair with. Then the instructions (see appendix A for a translated sample) were read aloud.[18] Before the start of a sequence of ten auctions, subjects played three dry runs, where they knew that their partner was simulated by a pre–programmed strategy.[19] In the uniform–price sessions subjects were informed that after the first ten auctions, ten further auctions under a different rule would be conducted, without further details being given at that point. After all pairs had finished the first ten auctions, the instructions for the second part were again read aloud.

In the open auction formats the price stayed at 0 for four seconds and then increased at a rate of 1 per second. Bidders could drop out on one or both (if no bidder had dropped out before) units at any time. After one bidder dropped out on one unit and the other bidder was informed about this, the price stayed at the dropout level for four seconds and

[17]This is why we only played ten auctions per pair and auction type. We wanted the total number of periods not to exceed 20 to avoid subjects getting bored. We also wanted to keep the incentives in each auction relatively high with a limited budget.

[18]This implies that it was common knowledge that the bidders had flat valuations that were drawn independently from the same distribution.

[19]These pre–programmed strategies did not reflect any characteristics of the equilibria (in particular complete demand reduction in the uniform–price auctions) and the subjects were explicitly advised that they should not see these strategies as examples of a good or a bad strategy.

increased at a rate of 1 per second thereafter. If a bidder dropped out during these four seconds, the dropout was regarded as at the same price but later than the first dropout. At any time during the bidding process, the bidders could observe the current price, the number of items for sale and the number of active bids. The sealed–bid auctions were run in a straightforward way, i.e. both bidders simultaneously placed two bids. Subjects were informed that the order of the bids was irrelevant.

After each auction bidders were informed about the observed dropout prices in the open auctions, or all four bids in the sealed–bid auctions, as well as the resulting allocation, their own gains or losses and their accumulated profits.

The experimental software was developed in zTree (Fischbacher, 2007). The sessions lasted for about 60 to 80 minutes in the uniform–price auctions and for about 30 to 50 minutes in the other treatments. At the end of each session, experimental currency units were exchanged in real currency at a rate of DM 0.04 (Berlin) or CHF 0.04 (Zürich) per ECU. In addition subjects received DM 5 (Berlin) or CHF 10 (Zürich) as show–up fee.[20] Average total payoffs were 342 ECU in AA, 270 ECU in DA, 290 ECU in VA, 701 ECU in UPOS (350 in UPO and 351 in UPS), and 663 ECU in UPSO (312 in UPS and 351 in UPO). This resulted in average earnings (including show–up fees) of DM 25.23 (about EURO 12.90) in Berlin and CHF 27.29 (about EURO 17.75) in Zuerich.

4 Experimental Results

As stated above, in treatments UPOS and UPSO the subjects played both uniform–price auctions in sequence. For the general comparison of all five auctions we only consider the first set of auctions out of these sessions (denoted by UPO and UPS) since the behavior in the second set of auctions is not independent of the behavior in the first one. We analyze behavior in the second set of auctions (denoted by UPsO and UPoS) separately in subsection 4.7, looking in particular whether bidders move closer to the DR–equilibrium in

[20] In order to relate the earnings, the exchange rates are 1 CHF = 0.65 Euro and 1 DM = 0.51 Euro. Cost of living is higher in Zurich, which justified the higher returns. The higher show–up fee in Zurich is based on a longer average commute to the laboratory than in Berlin.

the sealed–bid auction if they played the open version first.

The scatter diagrams in figures 1.1 through 2.4 provide a first impression of the behavior of the bidders in the five different auctions. Figure 1.1. through 1.6 show the bids in the three different sealed–bid auctions, where "unit1 bids" refers to the (weakly) higher, and "unit2 bids" to the (weakly) lower bid of a bidder. Figures 2.1 through 2.4 show dropout prices in the open auctions, AA and UPO. "Double dropouts" are simultaneous dropouts of one bidder on both units. "First dropouts" are the first dropout of a bidder on a unit while "second dropouts" refer to the second dropout in one auction, i.e. the price where the auction ends, not necessarily to the second dropout of one bidder. While figures 2.1 and 2.3 depict the overall behavior, 2.2 and 2.4 depict the behavior of pairs that almost followed equilibrium behavior. In what follows we will refer to these figures in order to illustrate the results.

Below, we generally use non–parametric Mann–Whitney tests for comparisons between treatments. These are always based on aggregate data per pair. The aggregate is computed over all periods unless explicitly behavior in only the second five of the ten auctions is compared. For comparisons within a treatment (between the first five and the second five auctions or between the first and the second set of auctions in UPOS and UPSO), as well as for comparisons with equilibrium predictions, we generally use non–parametric Wilcoxon signed–rank tests, because the data are paired. Again the tests are based on aggregate data per pair.

4.1 Are First–Unit Bids Truthful in VA, UPO, UPS, and AA? (H1)

Result 1 (First–Unit Bids) *(i) In AA, UPO, and VA, a large fraction of first–unit bids resemble truthful bidding. Specifically, in AA and UPO more than half of the first–unit bids are within one ECU of the valuation and rarely (11% and 7%, respectively) exceed the valuation by more than one ECU.*[21] *Underbidding is more frequent than*

[21] In UPO and AA, we can observe first–unit bids only in a subset of auctions. We refer to shares of the observable bids here.

overbidding in both formats, with underbidding being statistically significant in UPO, but not in AA. In VA, about 40% of first–unit bids are within one ECU of the valuation while another nearly 40% exceed it by more than one ECU. Average bids, are however, less than one ECU above the valuation.

(ii) In UPS, in contrast, overbidding by more than one ECU is not only frequent (about one third of the bids), but also substantial (leading to average bids more than 5 ECU above the valuation). Still, overbidding is not statistically significant.

	Bid = Value	Bid = Value +/- 1 ECU	Bid > Value + 1 ECU
AA	53.0 % *	18.0 % *	11.0 % *
UPO	30.4 % *	21,4 % *	7.1 % *
VA	29.5 %	10.5 %	39.5 %
UPS	21.5 %	13.0 %	33.5 %

Table 1: Percentage of first–unit bids that are equal to the valuation, 1 ECU above or below, and more than 1 ECU higher than the valuation (*: observable bids).

Table 1 pinpoints the fraction of first–unit bids that were truthful or almost truthful in the four auction formats. Moreover, it gives information on the extent of overbidding.

Although overbidding is frequent in VA (see the table), in most of these cases, the degree of overbidding is small so that the average bid exceeds the valuation only slightly (0.78 ECU) and insignificantly ($p = 0.333$, Wilcoxon signed–rank test).

In contrast, in UPS overbidding is substantial (see Table 1). The average bid exceeds the valuation considerably (5.55 ECU) which just fails to be significant ($p = 0.114$). Two additional observations in UPS are interesting in this context: First, out of 72 instances where bidders overbid on at least the first unit, only 11 led to a loss for the bidder. This illustrates quite well that bidders in UPS hardly learn that overbidding is dominated. Moreover — quite surprisingly — only in one case a bidder revised his behavior after suffering a loss.

In UPO and AA bidders underbid their valuation on the first unit more frequently than they overbid. Relative underbidding on the first unit was not significantly different from 0 in AA (Wilcoxon signed–rank test, $p = 0.76$), but was significant in UPO ($p = 0.012$). Moreover, relative underbidding was significantly larger in UPO than in AA (Mann–Whitney test, $p = 0.016$).

4.2 Demand Reduction in UPO/UPS (H2)

Result 2 (Demand Reduction) *Complete demand reduction (bids of zero from bidders with valuations larger than zero) occurs substantially and significantly more frequently in UPO (33 cases) than in UPS (9 cases).*

Figures 2.3 and 2.4 show dropout prices in UPO.[22] Figure 2.4 shows that in three pairs both bidders almost always dropped out on one unit at price 0, independently of their valuation.[23] These three pairs almost played the DR–equilibrium strategy, while the other subjects either bid roughly consistent with the TT–equilibrium or do not seem to have followed a consistent strategy. In the DR–equilibrium rational behavior implies that even off the equilibrium path, whenever one bidder drops out on one unit, the other should immediately follow. This requirement was violated in 55 % of the observable cases.

Figure 1.2 shows the (weakly) lower bids in UPS ("unit2 bids"). As expected we observe substantially fewer cases of complete demand reduction. In particular, we observed only 9 zero–bids on the second unit (from bidders with positive valuations) in UPS, which were all placed by the same subject, while we observed 33 such bids in UPO (notably, in UPO, the number of zero bids increases from 12 in periods 1 to 5 to 21 in periods 6 to 10).[24]

[22] Recall that "second dropouts" refer to the second dropout in one auction, that is, the price where the auction ends, not necessarily to the second dropout of one bidder.

[23] Note that for the open auctions we can only include the observed bids in the Figures. For the unobserved bids, a lower threshold is given by the price at which the auction ended. Hence, the figures for the open auctions should not be directly compared to those for the sealed–bid auctions, because the latter show all bids, whereas the former show only the two lowest bids in each auction.

[24] In addition, there were 15 bids equal to 1 (by bidders with valuations larger than 1) in UPO, but only 4 in UPS.

The number of zero–bids is significantly higher in UPO than in UPS (Mann–Whitney test, $p = 0.085$). In UPS, only one subject consistently chose the TT–equilibrium strategy, which was, however, not part of an equilibrium either, since the other subject was underbidding on the second unit most of the time.

4.3 Equality of Bids and Bid Spreads (H3)

According to the equilibrium prediction, in AA, VA, and DA the bidders should place equal bids on both units. In this section, we study the deviation from this prediction and also compare it to the bid spreading in UPS, where this is consistent with equilibrium behavior.

Result 3 (Bid Spreading) *(i) Bid spreads are small in AA and VA, with about two thirds of observed bid spreads being smaller than 10% of the equilibrium bid.*

(ii) In sharp contrast with the equilibrium prediction, bid spreads are substantial and persistent in DA, with nearly half being larger or equal to 40% of the equilibrium bid. They are significantly larger than in VA, but statistically indistinguishable from those in UPS.

maxbid-minbid	UPO	UPS	AA	VA	DA
$= 0$	27%*	18%	64%*	49%	12%
$<$ 10% Equ.	43%*	34%	68%*	62%	15%
$\geq$ 40% Equ.	32%*	33%	18%*	14%	49%

Table 2: Share of bid pairs (*: of observable bid pairs) that are exactly equal, where the difference is smaller than 10, or larger than 40 percent of the (TT–) equilibrium bid.

In both AA and VA several subjects played exactly according to the equilibrium prediction. In AA, some bidders initially tried to cooperate by placing one extremely low bid, however, those attempts usually did not work out and were abandoned after a few rounds. For VA, the data for the individual bidders reveal that the hypothesis that both, the higher and the lower bid are drawn from the same distribution can be rejected at the 5% level for only 4 out of 20 bidders (Kolmogorov–Smirnov test). Equal bids in all 10 auctions were placed by 4 out of 20 bidders. Bid spreading decreased significantly over time according to a linear regression (with robust standard errors).

The most surprising result is obtained for DA, where according to the theoretical prediction, bid spreads should not occur. In sharp contrast, however, bids spreads were at the highest level among all treatments. First–unit bids were significantly higher (Wilcoxon signed–rank test $p = 0.021$) than the equilibrium bid while the average second–unit bid was (insignificantly) smaller than the equilibrium bid ($p = 0.139$). A Kolmogorov–Smirnov test shows that the hypothesis that both, the higher and the lower bids (relative to equilibrium bids) are drawn from the same distribution, can be rejected at the 5%-level for 12 out of 18 bidders. Hence, bid spreading (relative to equilibrium bids) was clearly more prominent in DA than in VA (Mann–Whitney $p = 0.0025$). We discuss possible explanations for this phenomenon in a companion paper (Grimm and Engelmann, 2005), where we analyze the DA data with respect to the bid–spreading in detail (which is beyond the scope of the present paper). Specifically, we reject risk aversion and misperception of probabilities as explanations and find support rather in favor of a joy of winning.[25] In UPS, where it is consistent with the DR–equilibrium, bid spreading (relative to equilibrium bids) was clearly larger than in VA (Mann–Whitney test, $p = 0.0082$), but indistinguishable from that in DA ($p = 0.807$).

4.4 Efficiency (H4)

Result 4 (Efficiency) *(i) Due to demand reduction, efficiency is lower in UPO than in the other auction formats, that do not differ substantially in terms of efficiency.*

(ii) Efficiency increases significantly over time in AA and hence in periods 6 to 10, efficiency in AA is significantly higher than in DA, UPO, and UPS.

In equilibrium both units are allocated to the bidder with the higher valuation in AA, VA, and DA, but only one unit in the DR–equilibrium of the uniform–price auctions. An

[25]While decreasing absolute risk aversion can yield moderate bid spreading, both bids would have to be higher than the risk neutral equilibrium bids, which we do not observe.

efficient allocation requires allocating both units to the bidder with the higher valuation, because independent of the price this maximizes social welfare (the sum of the bidders' profits and the auctioneer's revenue).

Because valuations are randomly and independently drawn in our experiment, simply comparing treatments with respect to the achieved total welfare would be biased by these random draws. We compare the auction formats with respect to three different efficiency measures that are aimed to minimize this bias:

- *Allocative Efficiency*: the number of efficiently allocated units (i.e. to the bidder with the higher valuation) relative to the total number of units.
- *Relative Efficiency Loss*: the loss in terms of total welfare relative to the maximum possible efficiency loss.
- *Relative Efficiency*: the achieved total welfare relative to maximum possible welfare.

Allocative efficiency does not reflect the actual magnitude of efficiency losses due to misallocations. If the "wrong" bidder obtains a unit, his valuation may be substantially or only slightly below the other bidder's valuation, causing either dramatic or small welfare losses. Our second and third measures take this into account. In Table 3 we report for each measure aggregate results over all pairs and periods, as well as separated into the first five periods and the second five periods.[26]

Concerning allocative efficiency over all periods, treatments do not differ much. None of the pair–wise differences is significant (Mann–Whitney, $p > 0.2$). In particular, in UPS the allocative efficiency was only slightly below that in AA, DA, and VA, although the predicted allocative efficiency in the DR–equilibrium is only half of that predicted in the three other auctions. In each of AA, VA, and UPS for exactly one pair all units were allocated efficiently. The low allocative efficiency in UPO is due to the coordination of some pairs on the DR–equilibrium. According to relative efficiency losses and relative efficiency, in UPO efficiency

[26]Aggregates are not averages over the relative measures, but relative measures computed with respect to aggregate data. For example, relative efficiency for one treatment corresponds to the total achieved welfare by all pairs in this treatment over all (respectively the first or second five) periods, divided by the aggregated maximum possible welfare. This minimizes the impact of outliers based on small valuations.

	Periods	allocative eff.	relative eff. loss	relative eff.
AA	**all**	**84 %**	**9.2 %**	**95.6 %**
	1 – 5	77 %	18 %	91.4 %
	6 – 10	91 %	0.8 %	99.6 %
VA	**all**	**82.5 %**	**13.8 %**	**93.3 %**
	1 – 5	77 %	16.8 %	92.4 %
	6 – 10	88 %	11.1 %	94.2 %
DA	**all**	**83.3 %**	**7.1 %**	**96.5 %**
	1 – 5	81.1 %	9.8 %	95.3 %
	6 – 10	85.5 %	4.5 %	97.8 %
UPS	**all**	**81 %**	**12.2 %**	**92.9 %**
	1 – 5	80 %	11.5 %	93.5 %
	6 – 10	82 %	12.9 %	92.2 %
UPO	**all**	**74 %**	**25.9 %**	**87.6 %**
	1 – 5	72 %	25.8 %	88.5 %
	6 – 10	76 %	25.9 %	86.6 %

Table 3: Efficiency, measured by allocative efficiency, relative efficiency loss, and relative efficiency.

is significantly lower than in both AA (Mann–Whitney, $p = 0.059$ resp. $p = 0.041$) and DA ($p = 0.086$ resp. $p = 0.06$).

Differences in efficiency are far more pronounced in the second five periods (see again Table 3). Efficiency increases over time are substantial only in DA, VA, and AA, and significant only in AA.[27] Indeed, efficiency in AA in the last five periods is significantly higher than in DA, UPO, and UPS.[28] None of the other treatments differ significantly at a 10% level with respect to any efficiency measure in periods 6 to 10.

A more detailed look at the efficiency losses in AA reveals two main underlying reasons for inefficient allocations. First, attempts to collude by demand reduction or by dropping at price 0 with both units in the case of low valuations, apparently with the hope of reciprocation in later periods. Second, situations where the valuations of both bidders were very close, so that small deviations from the equilibrium strategy could result in misallocations. Since attempts to collude are largely unsuccessful, misallocations due to the first reason have disappeared in the second half of the experiment. The remaining misallocations all lead to very small losses in terms of welfare. Consequently, whereas allocative efficiency reaches only 91 %, according to the two measures that account for the magnitude of efficiency losses, AA reaches almost perfect efficiency in the last five periods.

In contrast, in all other mechanisms, even in periods 6 to 10, the relative efficiency loss is larger than 4% and the relative efficiency below 98 %. The only auction format where allocative efficiency also increases substantially over time is VA. However, in contrast to AA, in VA it is not the misallocations with substantial efficiency losses that disappear over time. Hence while according to our overall results, none of the mechanisms appears to be clearly preferable in terms of efficiency, AA clearly is preferable in case of experienced bidders.

[27] $p = 0.039$, $p = 0.011$, and $p = 0.025$ (Wilcoxon signed–rank) for allocative efficiency, relative efficiency loss and relative efficiency, respectively; in all other treatments $p > 0.2$ with respect to each measure.

[28] For the comparison with DA we get $p = 0.054$, and $p = 0.045$ for relative efficiency loss and relative efficiency, respectively; with UPO we have $p = 0.073$, $p = 0.008$, and $p = 0.011$ for allocative efficiency, relative efficiency loss and relative efficiency, respectively; and with UPS, $p = 0.085$, $p = 0.004$, and $p = 0.004$ (same order, Wilcoxon signed–rank).

The disappearance of misallocations due to collusive attempts also suggests that AA would have proved superior in terms of efficiency if we had conducted more periods or with random matching, which makes collusion less likely. In UPO, in contrast, under random matching, one bidder might teach a series of other bidders the DR–equilibrium. Hence we suspect that the advantage of AA over UPO with respect to efficiency would even be larger under random matching than under fixed matching, so that the fixed matching employed in our experiment is a tougher test for the efficiency superiority of AA.

While efficiency also increases (though not significantly) over time in DA (and is second highest with respect to relative efficiency in periods 6 to 10), the misallocations in DA were primarily caused by bid spreading, a robust effect in most pairs (indeed it increases from the first five to the second five periods in five out of nine pairs). The increase in efficiency appears to be due primarily to the reduction in erratic behavior, but the remaining misallocations due to bid spreading are likely to persist even in experiments over substantially more periods. We suspect that increasing the number of bidders would emphasize the advantage of AA over DA. On the one hand, underbidding in attempts to cooperate is likely to decrease in AA for more than two bidders. On the other hand, if bid spreading in DA prevails for more bidders, the probability that first–unit bids of bidders with low valuation are higher than second–unit bids of bidders with high valuation (and hence for misallocations) increases in the number of bidders.

4.5 Auctioneer's Revenues (H5)

Result 5 (Revenue) *(i) Revenue equivalence is rejected for AA and DA, and it is also clearly rejected for the uniform–price auctions.*

(ii) Revenues are generally higher in sealed–bid than in open auctions.

The theoretical results predict equal expected revenues in equilibrium for VA, DA and AA. The empirical revenues (see Table 4), however, do not reflect this. The difference in revenues relative to the equilibrium revenue between AA and DA is significant (Mann–Whitney test, $p = 0.034$), but both do not differ significantly from the equilibrium[29] or

[29] Wilcoxon signed–rank test, $p = 0.139$ for DA and $p = 0.203$ for AA.

	% of Eq. Revenue	min %ER	max %ER
UPO	**68.97 %**	1.30 %	107.82 %
AA	**84.74 %**	43.73 %	115.55 %
VA	**95.58 %**	41.45 %	130.73 %
UPS	**106.74 %**	80.48 %	160.67 %
DA	**110.72 %**	83.13 %	145.29 %

Table 4: Percentage of (TT–) Equilibrium Revenue (%ER) reached in the different auctions. Min and max refer to the minimum and maximum among the aggregates per pair.

from VA.[30]

In the uniform–price auctions the (DR–) equilibrium revenues are 0. The empirical revenues were naturally higher. To compare the revenues in the two uniform–price auctions, in Table 4 we report revenues relative to the TT–equilibrium revenues (note that for the TT–equilibrium also revenue equivalence with the other three auction formats holds). The difference between UPO and UPS is substantial and significant (Mann–Whitney test, $p = 0.019$). Furthermore, the revenues were significantly smaller than in TT–equilibrium in UPO but not in UPS (Wilcoxon signed–rank tests, $p = 0.028$ and $p = 0.507$, respectively). Thus revenue equivalence, which would be implied if the same equilibrium is played in both auction formats, does not hold for the two uniform–price auctions either. This is, of course, consistent with result 2 that demand reduction is more frequent in UPO than in UPS. In line with the equilibrium prediction the relative revenues in UPO were significantly lower than in VA and DA (Mann–Whitney, $p = 0.070$ and $p = 0.006$, respectively), but the difference is not significant with respect to AA ($p = 0.290$).

4.6 Bidder Payoffs (H6)

Result 6 (Bidder Payoffs) *(i) In UPO and UPS bidder payoffs are significantly lower than the DR–equilibrium prediction.*

[30]Mann–Whitney, $p = 0.545$ for AA and $p = 0.165$ for DA.

(ii) In DA, bidder payoffs are significantly lower than in equilibrium, in UPO, and in AA.

	% DR–Eq. Bidder Profits	min % DR–EBP	max % DR–EBP
UPS	**68.75 %**	50.93 %	87.89 %
UPO	**67.83 %**	37.93 %	102.25 %

Table 5: Percentage of (DR–) Equilibrium Bidder Payoffs (DR–EBP) reached in UPO and UPS. Min and max refer to the minimum and maximum among the aggregates per pair.

	%Eq. Bidder Profits	min %EBP	max %EBP
AA	**107.40 %**	54.26 %	288.83 %
VA	**90.88 %**	43.36 %	164.01 %
DA	**82.37 %**	47.00 %	113.04 %
UPS	**83.02 %**	57.73 %	112.04 %
UPO	**108.02 %**	80.06 %	149.53 %

Table 6: Percentage of (TT–) Equilibrium Bidder Payoffs (%EBP) reached in the different auctions. Min and max refer to the minimum and maximum among the aggregates per pair.

Bidder profits in both, UPS and UPO, were significantly lower than the DR–equilibrium profits (see Table 5, Wilcoxon signed–rank test, $p = 0.007$ in UPO, $p = 0.005$ in UPS). In UPS profits were even lower than the TT–equilibrium profits (Table 6, Wilcoxon signed–rank test, $p = 0.059$) and profits relative to the TT–equilibrium payoffs were significantly smaller in UPS than in UPO (Mann–Whitney test, $p = 0.034$).[31]

In AA, attempts to collude during the first periods of the experiment resulted in payoffs that exceeded equilibrium payoffs in five pairs. However, average bidder profits are not significantly different from the equilibrium prediction (Wilcoxon signed–rank test, $p = 0.445$).

[31]The comparison between UPS and UPO yields different results depending on which equilibrium is used as a benchmark because the valuations were randomly drawn and hence different in the two treatments.

The extreme minimum and maximum profits (see Table 6) were partly a coincidence.[32]

In VA bidder profits are not significantly different from the equilibrium prediction (Wilcoxon signed–rank test, $p = 0.203$). For most pairs, profits were close to the equilibrium, with four pairs within 5% deviation of the equilibrium profits.

In DA, since average bids were above equilibrium, bidder profits were consistently lower than the equilibrium prediction in most pairs. Seven out of nine pairs were below 90% of the equilibrium profits. Bidder profits in DA were significantly lower than equilibrium profits (Wilcoxon signed–rank test, $p = 0.028$). Furthermore, profits in DA were significantly lower than in UPO (relative to the TT–equilibrium) and AA (Mann–Whitney test, $p = 0.028$ and $p = 0.041$, respectively).

4.7 Effects UPO ↔ UPS (H7)

We have seen above that the subjects learn to play the payoff dominant DR–equilibrium of the uniform–price auction better in the open version. They might, however, be able to transfer the demand reduction they learn in UPO to UPS, whereas playing UPS before UPO should not help them finding the DR–equilibrium in UPO. In order to check whether this intuition is right, we let the subjects who played UPO and UPS play another ten auctions in the other uniform–price format. We will refer to the ten open auctions that were played after the sealed–bid auctions as UPsO and to the sealed–bid auctions played after the open auctions as UPoS.

RESULT 7 *(i) Bidders who have played UPO first, play closer to the DR–equilibrium in the sealed–bid auction than those in UPS. They exhibit, however, less demand*

[32]For the "lucky" pair (that achieved 289% of equilibrium payoffs), in several auctions the valuations of both bidders in this pair were very close, so that the equilibrium payoffs were very small. Attempts to cooperate through demand reduction or generous dropping out at a low price with both units led to payoffs substantially above the equilibrium. The low bidder profits of 54% of equilibrium profits were also partly driven by a chance event. In this pair, the same bidder always had the lower valuation. This seems to have caused some frustration which resulted in overbidding, which may have been driven by spite or just by a desire to experiment. The other pairs' payoffs ranged from 91% to 138%.

reduction than they previously did in UPO.

(ii) There appear to be slight hysteresis effects for bidders who first play UPS and then the open auction. They exhibit less demand reduction than those who started with UPO.

	% TT–Eq. Revenue	min % TT–ER	max % TT–ER
UPS	**106.74 %**	80.48 %	160.67 %
UPoS	**71.33 %**	6.5 %	128.28 %
UPO	**68.97 %**	1.30 %	107.82 %
UPsO	**80.22 %**	34.15 %	102.37 %

Table 7: Percentage of (TT–) Equilibrium Revenues (% TT–ER) reached in UPO, UPS, UPoS, and UPsO. Min and max refer to the minimum and maximum among the aggregates per pair.

	% DR–Eq. Bidder Profits	min % DR–EBP	max % DR–EBP
UPS	**68.75 %**	50.93 %	87.89 %
UPoS	**71.03 %**	39.5 %	109.12 %
UPO	**67.83 %**	37.93 %	102.25 %
UPsO	**69.71 %**	34.96 %	96.66 %

Table 8: Percentage of (DR–) Equilibrium Bidder Payoffs (% DR–EBP) reached in UPO, UPS, UPoS, and UPsO. Min and max refer to the minimum and maximum among the aggregates per pair.

Three of the pairs that played UPO first cooperated almost from the start and continued to do so until the end of the open auctions. While those pairs realized roughly the DR–equilibrium profit most of the time in UPO, this did not carry over to the subsequent sealed–bid auctions with the same pricing rule.

Whereas there is no significant difference in the bidder profits relative to the DR–equilibrium between UPS and UPoS (see also Table 8), the auctioneer's revenue relative to the TT–equilibrium was much lower in UPoS than in UPS (see Table 7, Mann–Whitney test, $p = 0.013$). In UPoS revenue was also significantly lower than in the TT–equilibrium (Wilcoxon signed–rank tests, $p = 0.047$). Furthermore, efficiency was significantly lower in UPoS than in UPS (allocative efficiency 67.5% vs. 81%).[33] In addition, the number of extremely low (0 or 1) bids was significantly higher in UPoS than in UPS (Mann–Whitney test, $p = 0.047$). These results indicate that behavior got closer to playing the DR–equilibrium in the sealed–bid auction if the open auction was played first. The truthfulness of first–unit bids (see Table 9), as well as the aggregate bid spread, however, do not differ significantly between UPS and UPoS.

	High Bid = Value	High Bid = Value +/- 1	Low Bid $\in \{0, 1\}$
UPsO	46.7 % *	25.0 % *	36
UPO	30.4 % *	21.4 % *	48
UPoS	21.0 %	10.5 %	36
UPS	21.5 %	13.0 %	13

Table 9: Percentage of first–unit bids in the four uniform price autions that are equal to the valuation, and 1 ECU above or below the valuation and percentage of second–unit bids that are extremely low (0 or 1) (*: observable bids).

For the pairs that played UPS first and then UPsO most of the evidence is in line with the hypothesis that the DR–equilibrium is easier to learn in the open than in the sealed–bid format. Bidder profits relative to the DR–equilibrium in UPsO were on average close to those in UPO (see Table 8). Bidder profits relative to the TT–equilibrium increased significantly from UPS to UPsO ($p = 0.028$, Wilcoxon signed–rank test), as did the number of extremely low (0 or 1) bids on the second unit (Wilcoxon signed–rank test, $p = 0.079$), while the auctioneer's revenues decreased significantly (Wilcoxon signed–rank test, $p = 0.013$). On the other hand, difficulties in coordinating on the DR–equilibrium in UPS

[33] The difference is significant according to Mann–Whitney tests $p = 0.031$ for allocative efficiency, $p = 0.049$ for relative efficiency loss, $p = 0.096$ and for relative efficiency. Interestingly, the allocative efficiency was even slightly (but insignificantly) lower in UPoS than in UPO.

seems to have partially carried over to UPsO. For example, the number of extremely low bids is smaller than in UPO, while efficiency and auctioneer's revenues are higher (though none of these results is statistically significant). The allocative efficiency in UPsO is even larger than in UPoS (Mann–Whitney, $p = 0.072$) opposite to the efficiency comparison between UPS and UPO. Finally, bidders violated the requirement of the DR–equilibrium to drop out on one unit immediately once the other bidder had dropped out, more often in UPsO (66 % of the cases where it was possible). In UPO, it was violated in only 55 % of the cases. This does not seem to be attributable to a lower rationality of the bidders in UPsO than in UPO, because truthful bidding on the first unit was more frequent in UPsO than in UPO (see Table 9).[34]

4.8 Questionnaires

In all treatments there were participants who indicated in post–experimental questionnaires that they tried to cooperate, as well as participants who explicitly behaved competitively or even spiteful. There is no indication that subjects realized that demand reduction is an equilibrium in the uniform–price auctions. In the uniform–price auctions as well as in the Vickrey and Ausubel auctions, several subjects realized that complete demand reduction is (weakly) payoff dominating all (other) equilibria and some realized that in UPO cooperation is easier than in UPS, while none made an explicit reference to equilibrium logic. Many subjects cited avoiding losses as a primary aim or as a constraint on their attempts to maximize their payoffs.

5 Discussion

The results of our experiments are in line with some of the theoretical predictions, while they clearly contradict others. Demand reduction occurs in the uniform–price auctions, though it also does to a lesser extent in the Ausubel auction. The allocative efficiency is

[34] Alsemgeest et al. (1998) find similar hysteresis effects in open auctions. Subjects who play the open auction with multi–unit demand after playing the open auction with single unit demand exhibit substantially less demand reduction than those who played the multi–unit demand auction first.

lowest in UPO, and highest in AA, where the latter differs only slightly from UPS, VA, and DA over all periods with respect to the number of efficiently allocated units. The causes of misallocations, however, appear to be least robust in AA. As a consequence, efficiency is significantly higher in AA than in DA, UPO, and UPS in the second half of the experiment. The revenue equivalence of AA and DA is clearly rejected, as it is for the two uniform–price auctions. In clear contrast to the theory, the auctioneer's revenues do not primarily depend on the pricing rule, but whether the auction is open or sealed–bid. This confirms previous observations from the field (Klemperer, 2002b, 2004) and thus underlines the caution that is warranted in implementing open ascending uniform–price multi–unit auctions.

Some of the results do not come as a surprise, though not predicted by the equilibrium analysis. Overbidding is more frequent in UPS and in VA than in UPO and AA, apparently since in the sealed–bid auctions it is less clear that overbidding is dominated. Coordination on the DR–equilibrium seems to be much easier in UPO than in UPS, because one bidder can signal intentions to collude by dropping out. Bidding above the equilibrium strategy is much more frequent in DA than in VA, and in particular in AA, since in the latter cases this involves overbidding of the valuation, and it is easier to recognize that this is not optimal, than it is to calculate the optimal bids in DA. These behavioral effects cause the auctioneer's revenues to be higher in the sealed–bid auctions than in the open auctions.

Our primary results are qualitatively in line with those of Kagel and Levin (2001), though some differences apparently result from our design involving two human players. They also find more demand reduction in the uniform–price open auction than in the uniform–price sealed–bid auction. However, while they find much less demand reduction in the Ausubel auction, we find more extremely low bids in early stages of AA than in UPS. Apparently these extremely low bids were the results of attempts to collude, which is impossible in their design with simulated opponents.

In accordance with our results, Kagel and Levin (2001) also find much more overbidding in the uniform–price sealed–bid auction than in the two open auctions. Furthermore, in their experiment — as well as in ours — UPS yields higher revenues to the auctioneer but lower allocative efficiency than AA. Hence we provide some further indication for this theoretically unanticipated trade–off between revenue and efficiency in AA and UPS. We show that Kagel and Levin's main results do not seem to depend critically on the simulation of other participants by computers. In contrast to Kagel and Levin, in our experiment there seems to be surprisingly little learning both within and across auction rules (with the exception of AA where bidders learn that collusion does not work). Those subjects who manage to determine the equilibrium do so almost at once. This is particularly surprising given that our interactive environments seem to be more complicated and that we did not provide hints against overbidding.

In line with our observation that the pricing rule is less important for revenues than whether the auction is sealed–bid or open, List and Lucking–Reiley (2000) find little differences in revenues between VA and UPS. They also find more overbidding on the first unit in UPS compared to VA, as we do.[35] Our results also confirm the observation of List and Lucking–Reiley that the bid spreading is larger in UPS than in VA, and confirm that this leads to (slightly) more misallocations.

Surprisingly, we find that bid spreading is very strong in DA, where it is not consistent with equilibrium behavior. This seems to be caused by a myopic joy of winning which leads subjects to increase the probability of acquiring at least one unit per auction at the expense of expected profits. This phenomenon has no distorting effect in the other auction mechanisms, since the probability of acquiring at least one unit (without making losses) is maximized by bidding the valuation on the first unit, which is consistent with equilibrium behavior. We discuss the bid spreading in DA in more detail in a companion paper (Grimm and Engelmann, 2005) where we reject risk aversion and misperception of probabilities as explanations for the phenomenon. As we argue in our companion paper, our findings raise doubts about the adequacy of risk aversion as an explanation for overbidding also in single–unit first–price auctions. The reason is that the observed pattern (namely overbidding on the first unit) cannot be explained by risk aversion in multi–unit auctions. Explanations for similar phenomena, however, should be consistent across different auction formats (e.g., for single as well as multi–unit auctions).

We further observe that the total allocative efficiency is almost identical in VA and AA. Inefficient allocations in AA seem partially caused by bidders hoping that the second bidder

[35]In a related experiment, Engelbrecht–Wiggans, List and Reiley (2006), find that this effect disappears with 3 or 5 bidders. They also find, consistent with the theoretical prediction, that demand reduction is still present, but reduced if the number of bidders is increased.

will play the weakly dominated strategy of dropping out after a dropout of the first bidder (which the second bidder then sometimes does), whereas inefficient allocations in VA result from a higher number of bids that deviate from the valuation, though only slightly. The latter observation may possibly be due to the fact that in VA it is less transparent to the bidders that bidding their own valuation is dominant. Hence we find that the possibly more transparent mechanism in AA can compensate for the weaker equilibrium concept compared to VA, a finding in agreement with the results in Kagel et al. (2001). After some experience, though, the collusive attempts in AA are given up and efficiency is higher than in VA.

One interesting detail in our experiment is that statements in the post–experimental questionnaires are similar after the uniform–price auctions and after AA. Several participants tried to cooperate by reducing demand and they observed that this worked well in UPO, but less so in UPS and even less in AA. It seems, however, that no–one realized that cooperation was stable when it was an equilibrium. Hence, the equilibrium prediction organizes the data well for some pairs although the subjects do not think in these terms. This is, of course, interesting from a general perspective. Equilibria can yield good predictions even if they are possibly too sophisticated for subjects to determine, given that equilibrium choices can result from less sophisticated thought processes.

6 Conclusion

The results of our laboratory experiments — in a controlled setting — provide support for several important policy conclusions drawn from theoretical considerations and field observations. First, the uniform–price open auction achieves both the lowest revenues and the lowest efficiency due to theoretically predicted demand reduction.[36] Second, if the

[36] Goswami, Noe, and Rebello (1996) find that in a setting with a high number of bidders (11) and units (100) and identical valuations, in a uniform–price sealed–bid auction non–binding pre–play communication facilitates demand reduction whereas it shifts behavior towards the equilibrium in a discriminatory auction. This indicates that outside the laboratory, where communication is more likely, the differences with respect to efficiency between uniform–price and other auction formats may be stronger than in our (and others') results.

primary aim of the auction designer is efficiency, the Ausubel auction seems to be best suited, in particular if bidders have time to gain experience. This observation confirms the common perception among economists that open ascending auctions have beneficial effects due to greater transparency. Third, if the focus is on revenues, sealed–bid auctions perform best due to frequent overbidding in the uniform–price sealed–bid and Vickrey auctions and to bids generally exceeding equilibrium bids in the discriminatory auction.

Finally, while we do find evidence for demand reduction in uniform–price auctions, we also find substantial deviations from the equilibrium prediction in the discriminatory auction, which leads to efficiency losses. Our findings strongly suggest that ongoing debates whether uniform–price or pay–as–bid auctions are preferable (i.e. in electricity markets) cannot be decided mainly on the basis of theoretical arguments.

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A Instructions (Ausubel Auction) — Not for Publication

Please read these instructions carefully. If there is something you do not understand, please raise your hand. We will then answer your questions privately. The instructions are identical for all participants.

In the course of the experiment you will participate in 10 auctions. In each auction you and another bidder will bid for two units of a fictitious good. This other bidder will be the same in each auction. Each unit that you acquire will be sold to the experimenters for your private resale value v. Before each auction this value **per unit**, v, will be randomly drawn independently for each bidder from the interval $0 \leq v \leq 100$ ECU (Experimental Currency Unit). Any number between 0 and 100 is equally probable. The private resale values of different bidders are independent. **In each auction any unit that you acquire**

will have the same value for you. This value will be drawn anew before each auction.

Before each auction you will be informed about your resale value **per unit**, v. Each participant will be informed only about his or her own resale value, but not about the other bidder's resale value.

After a short break the auction starts:

The price **per unit** will be increased successively in steps of 1, beginning at a price of 0. At the beginning of the auction you are active on both units. At any time you can drop out on *one* unit by clicking the button *"dropout 1"* or you can drop out on *both* units simultaneously by clicking the button *"dropout 2"*.

If one of the bidders clicks the button *"dropout 2"*, the other bidder obtains both units for the price where the first bidder dropped out and the auction is finished (since then there are only two active bids left).

If one bidder drops out on one unit, the other immediately obtains one unit (since the first bidder has only one active bid left and can thus acquire at most *one* unit) for the price at which the first bidder dropped out.

Then the auction continues at the price at which the first unit was given away. Now only **one** unit is auctioned off and both bidders have only **one** active bid. If now one bidder drops out on this unit, the other bidder obtains this unit for the price at which the bidder dropped out and the auction is finished.

If upon reaching the maximal price of 100 ECU there are four active bids left, both bidders receive one unit for a price of 100 ECU. If upon reaching the maximal price of 100 ECU there is only one unit given away, (both bidders still have one active bid), then the other unit will be randomly allocated for a price of 100 ECU among the two bidders.

Your profit per unit acquired is your resale value minus the price at which you obtained the unit.

If you do not obtain a unit you neither receive nor pay anything. Hence your profit is 0.

Note that you can make losses as well. It is always possible, however, to bid in such a way that you can prevent losses for sure.

You will make your decision via the computer terminal. You will not get to know the names and code numbers of the other participants. Thus all decisions remain confidential.

One ECU corresponds to 0,04 DM. You will obtain an initial endowment of 5 DM. If you make losses in an auction these will be deducted from your previous gains (or from your initial endowment). You will receive your final profit in cash at the end of the experiment. The other participants will not get to know your profits.

If there is something you have not understood, please raise your hand. We will then answer your questions privately.

Scatter Diagrams, Sealed-Bid Auctions

Figure 1.1
UPS - Unit1 Bids

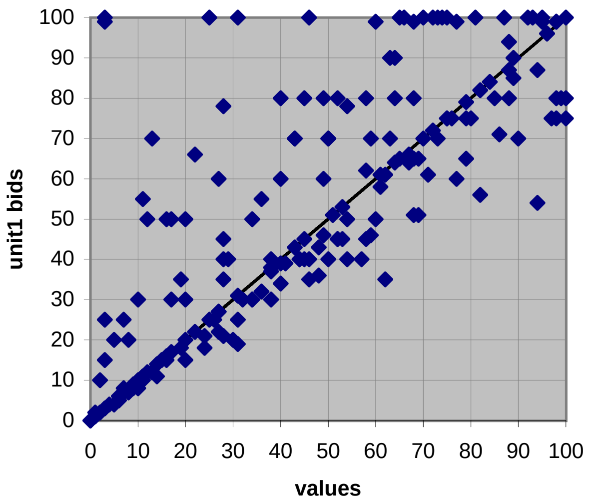


Figure 1.3
VA - Unit1 Bids

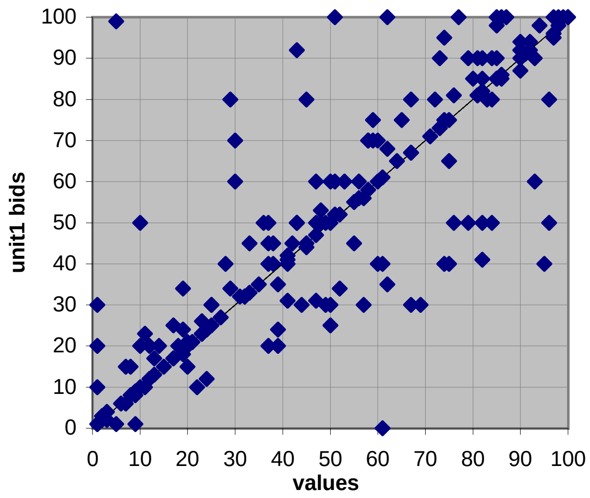


Figure 1.5
DA - Unit1 Bids

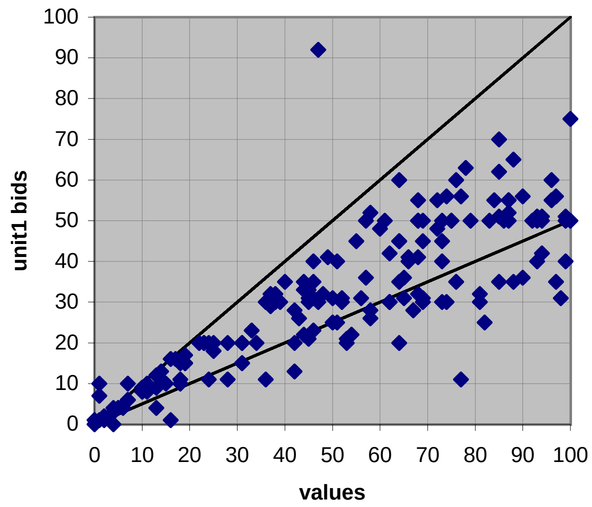


Figure 1.2
UPS - Unit2 Bids

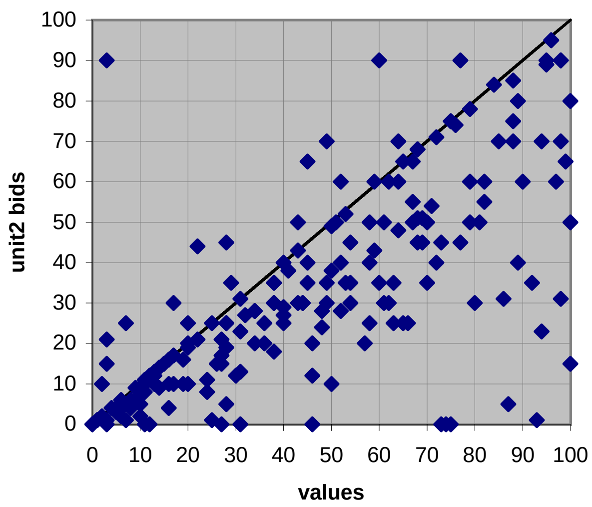


Figure 1.4
VA - Unit2 Bids

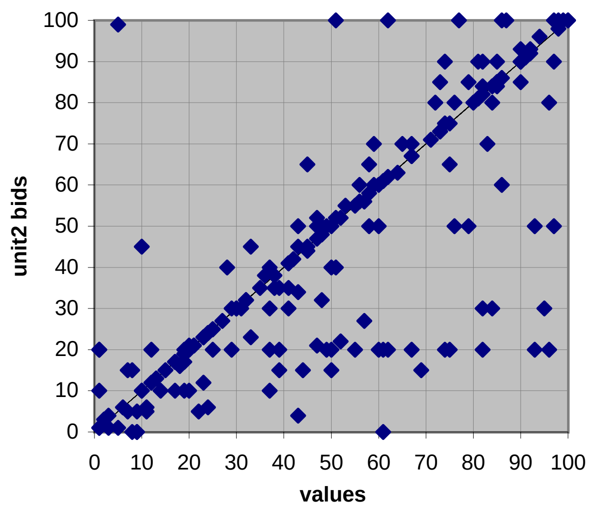
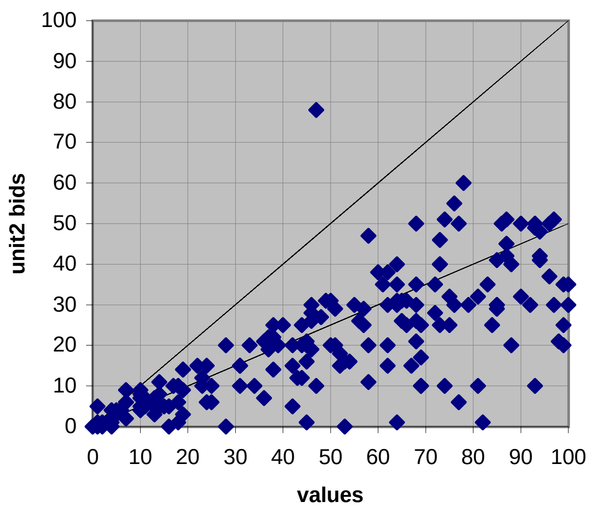


Figure 1.6
DA - Unit2 Bids



Scatter Diagrams - Open Auctions