Production under Uncertainty: A Characterization of Welfare Enhancing and Optimal Price Caps*

Veronika Grimm†
University of Erlangen–Nuremberg

Gregor Zoettl‡
University of Cologne

July 23, 2008

Abstract

We analyze the effect of price caps on equilibrium production and welfare in oligopoly under demand uncertainty. We find that high price caps always increase production and welfare as compared to the situation without price cap. Price caps close to marginal cost may lead to zero production, depending on the nature of uncertainty. We characterize the optimal price cap and show that typically, the optimal price cap is bounded away from marginal cost.

Keywords: Demand uncertainty, Cournot competition, price caps
JEL classification: D43, L13, D41, D42, D81.

---

*We thank Rabah Amir, Claude d'Aspremont, Gary Bolton, Estelle Cantillon, Patrik Legros, Benny Moldovanu, Axel Ockenfels, Harald Uhlig, and Achim Wambach as well as seminar participants at Berlin, Brussels, Bonn, and Cologne for helpful comments and suggestions and Felix Lamoroux for most valuable research assistance. Financial Support by the *Deutsche Forschungsgemeinschaft* is gratefully acknowledged.

†University of Erlangen–Nuremberg, Department of Economics, Lange Gasse 20, D–90403 Nürnberg, Germany, veronika.grimm@wiso.uni-erlangen.de

‡University of Cologne, Staatswissenschaftliches Seminar, Albertus-Magnus-Platz, D-50923 Köln, Germany, zoettl@wiso.uni-koeln.de

1 Introduction

In recent years price caps have been proposed (and also been used) in a variety of industries as a mean to combat the exercise of market power.[1] The classical literature on price regulation[2] provides the theoretical rationale: Consider a monopolist who faces a standard downward sloping demand curve. Compared to the situation without price cap, any price cap between the competitive and the profit maximizing price will increase his supply. By setting the price cap equal to marginal cost, the regulator can actually implement the welfare optimum. These results extend straightforwardly to the case of oligopolistic competition.

In practice, however, regulators typically do not know the firms' costs and also the firms face considerable uncertainty upon production, e. g. about future demand for their product. While the first problem has been investigated intensively in the literature[3], the issue of the firms' uncertainty about the future state of the world has not been paid much attention to up to date.[4] It is important to notice, however, that under demand uncertainty a low price cap (at or close to marginal cost) is typically suboptimal and may often even lead to a complete stoppage of production. The reason is that firms may face an *expected price* lower than marginal cost under such a regulatory regime.

Still, the common perception of practitioners is that price caps are an effective policy tool. Price caps indeed have been widely used, e.g. to discipline potential market power at the spot markets in the context of deregulating electricity markets. Typically, however, those price caps are well above marginal cost of the most costly operating unit in any demand scenario.[5] As we will show, relatively high price caps indeed increase production and welfare as compared to the situation without price cap.

In our paper we consider a standard Cournot game where demand is uncertain and firms choose their production quantities before the demand realization is known. The profit of each firm depends on the quantity produced and the price of the good, which is a function of total production in the industry and a parameter that captures demand uncertainty. Each firm has constant marginal cost c.[6] In the context of this model, we examine the introduction of a price cap $\bar{p} \geq c$.

[1]Price caps have first been proposed during the British Telecom privatization by Littlechild (1983), and have also widely been used in the electricity industry, see e.g. Stoft (2002).

[2]The literature on price regulation is very well developed for the regulation of monopolies. For a comprehensive overview see for example Laffont and Tirole (1993).

[3]See, e.g. Pint (1992), or Biglaiser and Riordan (2000).

[4]Cowan (2002) points out that price caps may negatively affect investment incentives in the presence of uncertainty. Dixit (1991), Dobbs (2004), and Roques and Savva (2006) analyze the impact of price ceilings in continuous time models with stochastic demand and indeed find that firms under–invest as compared to the competitive equilibrium.

[5]See, for example, Borenstein (2002), who discusses the use of price caps in the Californian electricity market.

[6]Our results can also be shown for convex cost. However, constant marginal cost allows a much clearer exposition.

We identify a range of high price caps that always lead to an increase of production and welfare as compared to the situation without price cap for any distribution of uncertainty. To gain some intuition, note that as compared to the situation without price cap, a high price cap (above the highest possible marginal revenue without a price cap) increases marginal revenue in any demand scenario where it is effective. Thus, it cannot introduce a disincentive to produce by decreasing expected marginal revenue. However, while any price cap in the range we identify yields higher production and welfare than no price cap, it requires exact knowledge of the distribution to pin down the optimal price cap.

When it comes to low price caps our results are ambiguous. Low price caps may have negative effects on production and welfare, and a price cap close to marginal cost may even lead to a complete stoppage of production. Thus, typically the optimal price cap is bounded away from marginal cost. However, optimality of a price cap arbitrarily close to marginal cost is also possible under demand uncertainty. We conclude that the attractiveness of low price caps depends on the exact distribution of uncertainty.

Note that firms which *produce* under demand uncertainty may have an incentive to withhold some of their production in case demand turns out to be low, in order to affect prices. This can be analyzed as a two stage game where firms first choose production under demand uncertainty and then decide on sales once uncertainty unraveled. We show that in this market game with free disposal all of the above conclusions (for the case without free disposal) continue to hold.[7] Moreover, we are able to pin down the lower bound of the interval of welfare increasing (high) price caps to be the competitive price given the highest demand realization. This is a valuable policy result, since in those markets where detailed information on cost is available (as, for example, the electricity market) this price cap can easily be calculated whereas almost nothing is known for the case of market power.

The paper is organized as follows: In section 2 we introduce the model. Section 3 contains our results. In section 3.1 we show existence, characterize the equilibrium, and provide comparative statics results for low and high price caps. In 3.2 we show existence and uniqueness of equilibrium in the free disposal case and show that qualitatively the same conclusions can be drawn as in section 3.1. Section 3.3 characterizes the optimal price cap. In a discussion (section 4) we finally illustrate our results with a numerical example. Section 5 concludes.

[7] This model, which we analyze in section 3.2, can also be interpreted as a game where firms first invest in capacity under demand uncertainty and then produce after uncertainty unraveled. Our analysis then shows that in this two stage game a high price cap would increase equilibrium *investment* and welfare as compared to the situation without price cap.

2 The Model

We consider a market game where n symmetric firms simultaneously produce a homogenous good at constant marginal cost c.[8] Market prices are subject to a price cap $\bar{p}$. Denote by $q = (q_1, \ldots, q_n)$ the vector of outputs of the n firms, and let $Q = \sum_{i=1}^{n} q_i$ be total quantity produced in the market. Market demand is given by a function $P(Q, \theta)$, which depends on total quantity and a random variable θ, which represents the (uncertain) "demand scenario". Denote by $[\underline{\theta}, \overline{\theta}]$ the range of possible demand scenarios and by $F(\theta)$ the probability distribution of θ, with the corresponding density $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. We introduce the parameter $z \leq 0$ as a lower bound on market prices in order to take into account non-negativity of prices ($z = 0$) or disposal cost ($z < 0$) and denote the quantity where this lower bound is met by $\bar{Q}(\theta)$.[9] The following regularity assumptions on demand have to be satisfied only for quantities $Q < \bar{Q}(\theta)$:

Assumption 1 *(i) $P(Q, \theta)$ is continuously differentiable*[10] *in Q with*[11] *$P_q(Q, \theta) < 0$ and $\lim_{Q \to \infty} P(Q, \theta) < c$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.*

(ii) $P(Q, \theta)$ is differentiable in θ with $P_\theta(Q, \theta) > 0$ $\theta \in (\underline{\theta}, \overline{\theta}]$.

In our model, firms decide on production before the realization of θ is known. The expected profit from operating in a market with price cap $\bar{p}$ if production is q is given by

$$\pi_i(q, \bar{p}) = \int_{\underline{\theta}}^{\overline{\theta}} \min\{\bar{p}, P(Q, \theta)\} q_i(\theta)\, dF(\theta) - c q_i. \tag{1}$$

Throughout the paper we consider only those cases where, in absence of a price cap, production is *gainful*, i. e. $\mathbf{E}(P(0, \theta)) > c$. We are interested in pure strategy Nash equilibria of the game. For any fixed price cap $\bar{p}$ we denote an equilibrium by $q^*(\bar{p})$ and the corresponding total equilibrium output by $Q^*(\bar{p})$. Since a price cap below marginal cost c leads to no production, we restrict ourselves to price caps $\bar{p} > c$.

3 Results

This section contains all our results. When we analyze welfare effects we will refer to total welfare (W) and consumer welfare (CW), which is calculated under the assumption of

[8]The assumption that marginal cost is constant is made for easier exposition. All the results can be shown to hold also for increasing marginal cost, however, with much higher technical effort.

[9]In case the lower bound is not binding we can set $\bar{Q}(\theta) = \infty$.

[10]Differentiability is not crucial for our results but makes exposition easier.

[11]Throughout the paper we denote the derivative of a function $g(x, y)$ with respect to an argument z, $z = x, y$, by $g_z(x, y)$, the second derivative with respect to that argument by $g_{zz}(x, y)$, and the cross derivative by $g_{xy}(x, y)$.

efficient rationing of consumers.[12]

3.1 Equilibrium Analysis and Comparative Statics

In this section we characterize equilibrium production in the Cournot market game under demand uncertainty. Moreover, we provide comparative statics results for low and high price caps.

Theorem 1 (Equilibrium with Price Caps) *For any price cap $\bar{p}$ the Cournot market game with uncertain demand has no asymmetric equilibria and at least one symmetric equilibrium. In any equilibrium with positive production expected marginal revenue equals marginal cost, i.e.*[13]

$$Q^* = \left\{ Q : \int_{\theta^Q(q)}^{\theta^{\bar{p}}(q,\bar{p})} \left[P(Q,\theta) + P_q(Q,\theta)\frac{Q}{n} \right] dF(\theta) + \int_{\theta^{\bar{p}}(q,\bar{p})}^{\overline{\theta}} \bar{p} dF(\theta) = c \right\}, \qquad (2)$$

where $\theta^Q \in [\underline{\theta}, \overline{\theta}]$ is the demand scenario from which on production is binding[14] *and $\theta^{\bar{p}} \in [\underline{\theta}, \overline{\theta}]$ is the demand scenario where the price cap is met.*

Proof See appendix A. □

Before we provide some intuition, we make the following definitions that simplify the discussion of our results.

Definition 1 *Denote by $\bar{\rho}^0$ the highest price cap that yields zero production and by $\bar{\rho}^\infty$ the lowest non-binding price cap in the highest equilibrium (denoted $Q^{*h}(\bar{p})$), i.e.*

$$\bar{\rho}^0 = \left\{ \bar{p} : \int_{\underline{\theta}}^{\tilde{\theta}^{\bar{p}}(0,\bar{p})} P(0,\theta)dF(\theta) + \int_{\tilde{\theta}^{\bar{p}}(q,\bar{p})}^{\overline{\theta}} \bar{p}dF(\theta) = c \right\}, \quad \bar{\rho}^\infty = \left\{ \bar{p} : \bar{p} = P(Q^{*h}(\bar{p}), \overline{\theta}) \right\}.$$

Denote by Q^0 welfare optimal production in the lowest demand scenario $\underline{\theta}$, i.e.

$$Q^0 = \max\{0, (Q : P(Q, \underline{\theta}) = c)\}.$$

Price caps higher than $\bar{\rho}^\infty$ are never binding in the highest equilibrium. In the corresponding first order condition it holds that $\theta^{\bar{p}}(q, \bar{\rho}^\infty) = \overline{\theta}$, and the second term on the LHS of (2) has to be dropped. The resulting first order condition has a straightforward intuition: The LHS is expected marginal revenue, whereas the RHS is just marginal cost of production. Note that on the LHS expectation is taken only over those demand scenarios where the additional unit produced would actually generate nonzero marginal revenue (i. e. the

[12] I.e., $W = \mathbf{E}_\theta \left[\int_0^Q P(x,\theta)dx - cQ \right]$ and $CW = \mathbf{E}_\theta \left[\int_0^Q (P(x,\theta) - \min\{P(Q,\theta), \bar{p}\})dx \right]$.

[13] In the following we set $z = 0$, which means that prices are nonnegative. This is the most natural case. In the appendix, a characterization is provided for general values of z.

[14] That is, $\theta^Q = \{\theta : P(Q,\theta) = 0\}$. Note that here we have set $z = 0$ for easier exposition. For z low enough it would always hold that $\theta^Q = \underline{\theta}$.

lower limit of integration is $\theta^Q(q)$, not $\underline{\theta}$). The reason is that only production of a unit that generates a nonzero price is relevant for the firms' production decisions.[15]

Now consider the case of a binding price cap and positive production. The corresponding first order condition again equates expected marginal revenue of production with marginal cost. As in the case without binding price cap, when calculating expected marginal revenue, the firm considers only those demand scenarios that generate positive prices (i.e. integration starts at θ^Q). Marginal revenue in scenario θ is the standard expression until the price cap is binding at $\theta^{\bar{p}}$, from there on, marginal revenue equals the price cap.

As it has become clear, in all cases with positive production, the first order condition equates expected marginal revenue of production with marginal cost. Notice that theorem 1 covers degenerate uncertainty as a special case. Then, it holds that $\underline{\theta} = \theta^Q = \theta^{\bar{p}} = \overline{\theta}$ and, as $\bar{p}$ approaches marginal cost c, production is always positive (and maximized).[16] In particular, as compared to the situation without price cap, a binding price cap $\bar{p} > c$ implies for the corresponding equilibrium that (i) production weakly increases, and (ii) total welfare weakly increases. We now investigate in how far those conclusions continue to hold under genuine demand uncertainty. The following theorem characterizes an interval of (high) binding price caps which, *without any further assumptions on the distribution* $F(\theta)$, can be classified as production and welfare enhancing as compared to the situation without price cap.

Theorem 2 (High Price Caps) *Denote the equilibrium with the highest production by Q^{*h} and by $\overline{MR}$ the highest marginal equilibrium revenue without binding price cap*[17]. *In this equilibrium, a (binding) price cap $\bar{p} \in \left[\overline{MR}, \bar{p}^{\infty}\right]$ increases equilibrium production, consumer surplus and total welfare, and lowers average prices as compared to the market game without binding price cap.*

Proof See appendix B. □

Given the equilibrium characterization in theorem 1, the intuition is straightforward: Any price cap above the highest possible marginal equilibrium revenue without price cap increases marginal revenue in *all* scenarios where it is binding (compared to the situation without price cap). In the new equilibrium with binding price cap expected marginal revenue must again be equal to marginal cost. This can only be achieved by decreasing marginal revenue in scenarios with non–binding price cap, i.e. by increasing production.

Notice that even in cases where the equilibrium is unique we cannot make statements about monotonicity, in particular we cannot characterize the best price cap in $\left[\overline{MR}, \bar{p}^{\infty}\right]$

[15] Recall that we set $z = 0$ in the theorem. If $z < 0$, additional production yields a negative marginal revenue equal to z in low demand scenarios, which would also appear in the first order condition. See the appendix, where this more general case is analyzed.

[16] Note that we assumed that production is gainful, i.e. $P(0) > c$ in case of certain demand.

[17] I.e. $\overline{MR} = \max_\theta\{P(Q^{*h}(\bar{p}^{\infty}), \theta) + P_q(Q^{*h}(\bar{p}^{\infty}), \theta)\frac{Q^{*h}(\bar{p}^{\infty})}{n}\}$

without further assumptions on the distribution.[18] Our result implies that the standard comparative statics predictions (i.e. a price cap increases production and welfare as compared to the situation without price cap) always hold for relatively high (but binding) price caps also if demand is uncertain (i.e. not only for deterministic demand). We conclude that high price caps are always desirable from a welfare point of view and that identification of the best price cap in the specified interval obviously requires precise information on the demand distribution.

For low price caps, on the contrary, this conclusion cannot generally be drawn. As it is easily seen, if it holds that $P(0, \underline{\theta}) < c$, optimal production must be zero for price caps close to marginal cost. The reason is that any strictly positive production would yield expected marginal revenue lower than marginal cost. Thus, the lowest price cap that leads to positive production may be bounded away from marginal cost, i. e. $\bar{\rho}^0 > c$. If, on the contrary, it holds that $P(0, \underline{\theta}) > c$ (i. e. positive production is welfare optimal even in the lowest demand scenario), equilibrium production is positive as $\bar{p} \searrow c$. Note that for $\bar{p} \searrow c$ it must hold that $\underline{\theta} = \theta^Q = \theta^{\bar{p}}$: Only if the price cap is binding from the lowest demand realization on, it is guaranteed that the price never drops below c, i. e. that the firm makes no losses in expectation. In other words, whenever equilibrium production is positive as $\bar{p} \searrow c$, it coincides with the perfectly competitive quantity in the lowest demand scenario, Q^0.[19]

In the following theorem we show that in general production can increase or decrease in the price cap at price caps close to marginal cost. We moreover provide monotonicity results[20] under a standard regularity assumption.

Theorem 3 (Price Caps Close to Marginal Cost)

(i) if $P(0, \underline{\theta}) \leq c$, then production is zero for all $\bar{p} \in [c, \bar{\rho}^0]$ and production is increasing in $\bar{p}$ at $\bar{\rho}^0$.

(ii) Suppose that $P(0, \underline{\theta}) > c$ and $P_q(Q^0, \theta) + P_{qq}(Q^0, \theta)\frac{Q^0}{n} < 0$ for all θ. Then, total production as $\bar{p} \searrow c$ equals Q^0, and output is decreasing (increasing) in $\bar{p}$ if

$$-\frac{P_q(Q^0, \underline{\theta})}{P_\theta(Q^0, \underline{\theta})} \frac{Q^0}{n} f(\underline{\theta}) > (<) 1.$$

Proof See appendix C. □

[18]We address this issue in theorem 7.

[19]Compare also Earle et al. (2007), theorem 3 and corollary 2.

[20]Notice that we assume a continuous distribution of θ. If uncertainty follows a discrete distribution, then at a price cap close to marginal cost output is always decreasing in the price cap if, as $\bar{p} \searrow c$, production is positive. Earle et al. (2007) analyze price caps close to marginal cost for distributions F that are *continuously differentiable* on $\mathbb{R}$ and find that production is always increasing in $\bar{p}$ for $\bar{p}$ sufficiently close to c. Our theorem 3 shows that their result does not always hold in the class of continuous distributions.

From a policy perspective theorem 3 illustrates that price caps relatively close to marginal cost may be desirable even under uncertain demand.[21] We conclude that while in many plausible cases both, production and welfare are zero or almost zero for price caps close to marginal cost, depending on the distribution of θ, a price cap close to marginal cost may even be optimal. Note, however, that even though a price cap at marginal cost may be optimal, under demand uncertainty the welfare optimum can never be implemented by any price cap.[22]

3.2 Free Disposal

We now consider the case that uncertainty is resolved after the firms' production decisions but before they decide on the quantities they want to sell. Firms may benefit from strategic withholding in particular if demand turns out to be low. We assume that disposal is costless.[23]

In the presence of free disposal we have to analyze a two stage game. At the first stage firms decide on production quantities $q(\bar{p}) = (q_i(\bar{p}))_{i=1,\ldots,n}$. Then, they learn the true state of the world, θ. Once they know the state of the world, they decide on the quantities $y(q,\bar{p},\theta) = (y_i(q,\bar{p},\theta))_{i=1,\ldots,n}$ they want to sell. We assume that in case the price cap is binding the firms' sales are as equal as possible.[24] We make the following additional regularity assumptions that have to be satisfied for quantities $Q < \bar{Q}(\theta)$:

Assumption 2 *(i) $P(Q,\theta)$ satisfies $P_q(Q,\theta) + P_{qq}(Q,\theta)q_i < 0$ for all $\theta \in [\underline{\theta},\overline{\theta}]$.*

(ii) $P(Q,\theta)q_i$ is (differentiable) strict supermodular in q_i and θ, i. e. $\frac{d^2[P(Q,\theta)q_i]}{dq_i d\theta} > 0$ for all i, θ, and q_{-i}.[25,26]

We get the following result:

Theorem 4 (Equilibrium with Free Disposal) *For any price cap $\bar{p} > c$, the Cournot market game with free disposal has a unique equilibrium $q^*(\bar{p})$ which is symmetric. If equilibrium production is positive it is uniquely characterized by*

$$Q^*(\bar{p}) \quad = \quad \left\{ Q : \int_{\tilde{\theta}^Q(q,\bar{p})}^{\tilde{\theta}^{\bar{p}}(q,\bar{p})} \left[P\left(Q,\theta\right) + P_q\left(Q,\theta\right)\frac{Q}{n} \right] dF\left(\theta\right) + \int_{\tilde{\theta}^{\bar{p}}(q,\bar{p})}^{\bar{\theta}} \bar{p} dF(\theta) = c \right\}, \tag{3}$$

[21]See also theorem 7, where we show that if F has an increasing hazard rate, $\bar{p} \searrow c$ is actually optimal if $q^*(\bar{p})$ is decreasing for $\bar{p} \searrow c$.

[22]We show this formally in the proof of theorem 2, in lemma 1.

[23]We do so mainly for easier exposition. All our results continue to hold if we assume additional cost of selling (which could also be negative).

[24]Note that this does not imply that no asymmetric equilibria of the game exist. It does imply, however, that *if* firms *produce* equal quantities, they always *sell* equal quantities if the price cap is binding.

[25]Throughout the paper q_{-i} denotes the quantities produced by the firms other than i, and $Q_{-i} = \sum_{j \neq i} q_j$.

[26]Part (ii) of the assumption is not essential. It makes it, however, much easier to write down expected profits.

where $\tilde{\theta}^{Q}(q,\bar{p}) \in [\underline{\theta},\overline{\theta}]$ *is the demand scenario from which on firms sell their entire production and* $\tilde{\theta}^{\bar{p}}(q,\bar{p}) \in [\underline{\theta},\overline{\theta}]$ *is the demand scenario where the price cap starts to be binding and firms sell their entire production.*

PROOF See appendix D. □

Note that the first order condition in theorem 4 follows the same intuition as in the case without free disposal. However, unlike in theorem 1, we can establish uniqueness for all possible price caps (not only for $\bar{p} \searrow c$ as in the case without free disposal).

Analogously to the case without free disposal (theorems 2 and 3), we obtain the following comparative statics results concerning the effect of price caps on equilibrium production, average prices, and welfare.

THEOREM 5 *The statements of theorems 2 and 3 remain valid for the modified game with free disposal.*

PROOF See appendix E □

In the free disposal case, we moreover find that under relatively mild assumptions on the demand function[27] the highest possible market price reached under perfect competition could lend itself as a welfare enhancing price cap:

THEOREM 6 (PRICE CAP AT THE HIGHEST COMPETITIVE PRICE) *Suppose demand can be decomposed such that* $P(Q,\theta) = a(\theta) + b(\theta)\widetilde{P}(Q)$ *and denote the perfectly competitive price in scenario* θ *by* $p^{PC}(\theta)$. *A price cap* $\bar{p} = p^{PC}(\overline{\theta})$ *always increases equilibrium production, consumer surplus and total welfare, and lowers average prices as compared to the market game without binding price cap.*

PROOF See appendix F. □

Note that from a policy maker's perspective the result is actually very interesting. For example in electricity markets[28], regulators typically have quite detailed information on marginal cost. Thus, calculating $p^{PC}(\overline{\theta})$ by modeling the *competitive benchmark* seems relatively easy, while basically nothing is known for the case of market power. The theorem nicely connects the two scenarios of perfect competition and oligopoly, giving a clear cut policy result.

3.3 Optimal Price Caps

As we have shown in sections 3.1 and 3.2, we can always identify a range of binding price caps that are desirable from a welfare point of view, independently of the distribution of θ. This

[27]These do not seem necessary, but they allow for a very intuitive proof.

[28]Recall that with free disposal our model can also be interpreted as capacity choice under demand uncertainty prior to a production decision once demand is known, compare Grimm and Zoettl (2006).

finding clearly calls for a more precise characterization of optimal price caps, which is the aim of this section. We focus on the case of free disposal, since the established uniqueness result allows for a more clear cut analysis. Whenever in the case without disposal, given $P(Q,\theta)$ and c, the equilibrium is unique for all $\bar{p}$, our results apply to both cases.

In the following theorem we characterize the price cap that maximizes equilibrium production. In order to do so we need to make regularity assumptions on the distribution of θ.[29]

Theorem 7 (Optimal Price Cap) *Suppose that demand can be decomposed such that* $P(Q,\theta) = \theta + \widetilde{P}(Q)$ *and that the hazard rate* $h(\theta) := \frac{f(\theta)}{1-F(\theta)}$ *is increasing.*[30] *Then there exists a unique price cap* $\bar{p}^*$ *which maximizes production. It holds that* $\bar{p}^* = c$ *if production is decreasing at* $\bar{p} = c$. *Otherwise* $\bar{p}^*$ *is uniquely characterized by*

$$-P_q(Q^*, \tilde{\theta}^{\bar{p}}) \frac{Q^*}{n} = \frac{1 - F(\tilde{\theta}^{\bar{p}})}{f(\tilde{\theta}^{\bar{p}})}. \tag{4}$$

If it holds that $\bar{p}^* = c$, *then* $\bar{p}^*$ *is the welfare maximizing price cap. If* $\bar{p}^* > c$, *it holds that* $\bar{p}^*$ *is an upper bound for the welfare maximizing price cap.*

Proof See appendix G. □

Let us point out the trade off that has to be solved at the production maximizing price cap. Recall from theorems 1 and 4 that firms choose the quantity that just equates expected marginal revenue of production with marginal cost. Lowering a given price cap affects expected marginal revenue and thus, the firms' production decisions. We observe two effects: On the one hand, lowering the price cap by an increment decreases marginal revenue in all scenarios where the price cap has been binding. The expected marginal loss equals $[1-F(\tilde{\theta}^{\bar{p}})]$. On the other hand, the price cap becomes binding also in lower demand scenarios and thereby marginally increases expected revenue by $-P_q(Q^*, \tilde{\theta}^{\bar{p}})q_i f(\tilde{\theta}^{\bar{p}})$.[31] Condition (4) as stated in theorem 7 balances those two effects. Note that the first effect lowers production incentives, while the second effect encourages production. At high price caps, the first effect is necessarily small (since the range of demand scenarios for which the price cap is binding is small), such that the second effect always dominates. Note that it depends on the exact distribution of θ whether the production–maximizing price cap lies in $\left[\overline{MR}, \bar{p}^\infty\right)$, or below. In particular, corner solutions where $\bar{p}^*$ is arbitrarily close to c are also possible. We illustrate our findings with a simple example in section 4.

[29] Recall that in theorem 2 and 5, we could not make statements about monotonicity without further assumptions on the distribution.

[30] In order to prove the theorem it is sufficient to assume $P_{q\theta} = 0$, such that demand can be decomposed such that $P(Q,\theta) = v(\theta) + \widetilde{P}(Q)$. Then, the hazard rate of the transformed random variable $v(\theta)$ would have to be increasing. In the theorem we present a slightly less general statement for easier exposition.

[31] Note that marginal revenue in scenario θ without binding price cap is $P_q(Q^*,\theta)q_i + P(Q^*,\theta)$ and thus, a binding price cap $\bar{p} = P(Q^*, \tilde{\theta}^{\bar{p}})$ increases marginal revenue in scenario $\tilde{\theta}^{\bar{p}}$ by $-P_q(Q^*, \tilde{\theta}^{\bar{p}})q_i > 0$.

Let us also discuss the welfare implications of our results. In the case without free disposal, the production maximizing price cap always maximizes welfare. Furthermore, if free disposal is possible, but firms do not have an incentive to withhold in any demand scenario[32], the price cap characterized in theorem 7 also maximizes welfare. In particular, this holds true for any corner solution where $\bar{p}^* \searrow c$. If, on the other hand, firms withhold a positive amount in equilibrium, the welfare maximizing price cap may be lower, but never higher than $\bar{p}^*$. The reason is that decreasing the price cap below $\bar{p}^*$ on the one hand decreases production but, on the other hand, eliminates the incentive to withhold in additional demand scenarios such that the average quantity *sold* may increase and thus, also total and consumer welfare. Price caps higher than $\bar{p}^*$ lead to lower production without having the desirable effect on the firms' withholding decisions.

Let us finally briefly point out under what conditions the equilibrium is unique also in the case without free disposal, such that our results also apply in this case.[33] First of all, we obtain uniqueness whenever it holds that $\theta^Q = \underline{\theta}$ (in which case the expected profit is always concave). This holds true if *either* demand is not kinked (i.e. $\bar{Q}(\theta) = \infty$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$), *or* uncertainty is degenerate such that the price is above z even in the lowest demand scenario. However, also for kinked demand functions expected profits are quasiconcave if we impose restrictions on the distribution of θ (see our example in the next section). In all those cases where there exists a unique equilibrium, the results of this section also apply in the case without free disposal.

4 Discussion

In this section we illustrate our results both, in the free disposal and in the non–free disposal case, using a numerical example.

EXAMPLE 1 *Suppose demand is given by $P(Q,\theta) = \max\{0, \theta - Q\}$, where θ is uniformly distributed with support $[\underline{\theta}, 2]$. There are two firms with constant marginal cost $c = \frac{1}{2}$. In this particular example, the equilibrium is unique also in the case without free disposal. The first order condition is given by*

$$\int_{\theta^Q(q,\bar{p})}^{\theta^{\bar{p}}(q,\bar{p})} \left(\theta - \frac{n+1}{n} Q\right) \, dF(\theta) + \int_{\theta^{\bar{p}}(q,\bar{p})}^{\overline{\theta}} \bar{p} \, dF(\theta) = c,$$

where

$$\theta^{\bar{p}}(q,\bar{p}) = \min\left\{\overline{\theta}, Q + \bar{p}\right\}, \qquad \theta^Q(q) = \max\left\{\underline{\theta}, Q\right\}.$$

In the free disposal case the first order condition looks identical except for the fact that the

[32] That is, if $\tilde{\theta}^Q = \underline{\theta}$ (compare the discussion of theorem 4).

[33] The following discussion requires that part (i) of assumption 2 holds in both models.

limits of integration have to be substituted by

$$\tilde{\theta}^{\bar{p}}(q, \bar{p}) = \min\left\{\overline{\theta}, Q + \bar{p}\right\}, \qquad \tilde{\theta}^{Q}(q, \bar{p}) = \max\left\{\underline{\theta}, \min\left\{\tilde{\theta}^{\bar{p}}, \frac{n+1}{n}Q\right\}\right\}.$$

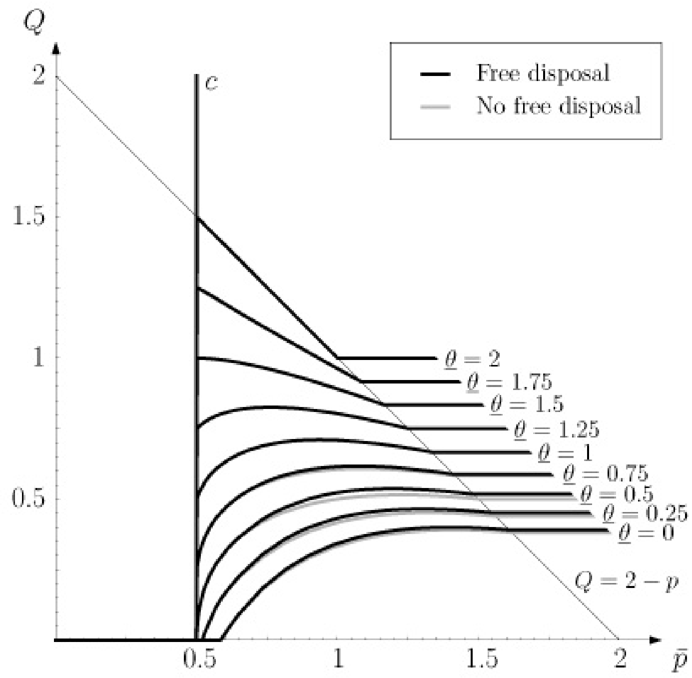

Figure 1: Production as a function of the price cap for various values of $\underline{\theta}$.

Figure 1 shows aggregate production as a function of the price cap, $Q^*(\bar{p})$, for several values of $\underline{\theta} \in [0, 2]$ for the free disposal case (black lines) and the case without free disposal (grey lines). As the graph illustrates, both cases look qualitatively identical. For all values of $\underline{\theta}$ and $\bar{p}$, it holds that production in the case without free disposal is lower than production under free disposal.[34] *This is due to the fact that production is less profitable if prices cannot be raised above zero unless the quantity is binding. Recall also that in the free disposal case, the welfare maximizing price cap might be below the price cap that maximizes production (but never above). For the case without free disposal, the welfare maximizing price cap coincides with the price cap that maximizes production. This implies that whenever $\bar{p} \searrow c$ maximizes production, it is also welfare optimal to choose that price cap in both cases.*

Notice that figure 1 nicely illustrates theorem 3 on low price caps (close to marginal cost): For $\underline{\theta} \in [0, 0.5)$, production is zero at price caps close to marginal cost (theorem 3, part (i)). If very low demand scenarios are not possible (i.e. for $\theta > 0.5$), production is positive at price caps close to marginal cost. Obviously, it depends on the exact nature of uncertainty whether production is increasing or decreasing in the price cap at this point (theorem 3, part (ii)).

[34]For $\underline{\theta} < 1.2$ production is higher under free disposal, for $\underline{\theta} \geq 1.2$ both cases collapse.

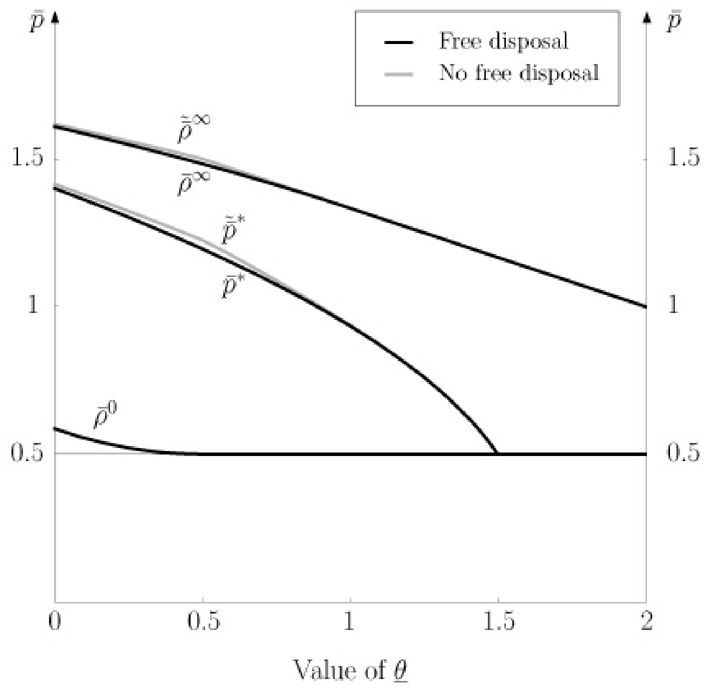

Figure 2: The optimal price cap $\bar{p}^*$, $\bar{\rho}^0$, and $\bar{\rho}^\infty$ as a function of $\underline{\theta}$.

Figure 2 shows for both cases the production maximizing price cap (that also maximizes welfare in the case without free disposal) as a function of $\underline{\theta}$. *Obviously, the optimal price cap is always below the lowest non–binding price cap,* $\bar{\rho}^\infty$, *and decreases in* $\underline{\theta}$, *as it is implied by theorems 2 and 7. For* $\underline{\theta} \in [1.5, 2]$, *a price cap* $\bar{p} \searrow c$ *maximizes production and welfare since, as it is easily seen from figure 1,* $Q^*(\bar{p})$ *is decreasing everywhere on* $(c, \overline{\theta}]$ *(compare the last part of theorem 7). Comparison with figure 1 moreover reveals that already for* $\underline{\theta} > 1.25$, *a price cap* $\bar{p} \searrow c$ *leads to higher production than no price cap (although the optimal price cap is still higher than c).*

In our example, the optimal price cap is always below the lower bound of the interval of desirable price caps we identify in theorem 2 (except for the case of deterministic demand, where the lower bound coincides with the optimal price cap). For low values of θ *the optimal price cap lies just outside the interval while for high values of* θ *it is considerably below the lower bound of the interval.*

5 Conclusion

In this article we have shown that in oligopoly under demand uncertainty, relatively high price caps are *always* welfare enhancing while low price caps decrease production and welfare in many genuine cases. We conclude that price caps "far away" from marginal cost but strictly below the lowest non–binding price cap are always desirable. As we have shown, this holds independently of whether the firms can dispose of produced quantity, or not. We have thoroughly investigated both cases and pointed out some important differences. For the case of free disposal we identified the competitive price under the highest possible demand realization to be a welfare enhancing price cap for any specification of uncertainty. This result is particularly appealing since the highest competitive price can be easily calculated if the regulator possesses information on marginal cost. We moreover characterized the optimal price cap and showed that, depending on the nature of uncertainty, it might be close to or well above marginal cost.

To summarize, our paper has shown that if we consider firms facing an uncertain demand function, standard comparative statics results qualitatively continue to hold for high price caps but not necessarily for low price caps. Contrary to the case of certain demand, however, for genuine uncertainty the welfare optimum cannot be reached by imposing the optimal price cap.

6 References

Amir, R. and V. Lambson (2000). On the Effects of Entry in Cournot Markets. *Review of Economic Studies*, 67, 235–254.

Biglaiser, G. and M. Riordan (2000). Dynamics of price regulation. *RAND Journal of Economics* 31, 744–767.

Borenstein, S. (2002). The Trouble with Electricity Markets: Understanding California's Electricity Restructuring Disaster. *Journal of Economic Perspectives*, 16, 191–211.

Dixit, A. (1991). Irreversible Investment with Price Ceilings. *Journal of Political Economy* 99 (3), 541–557.

Dobbs, I. (2004). Intertemporal price cap regulation under uncertainty. *Economic Journal* 114, 421–440.

Grimm, V. and G. Zoettl (2006). Capacity Choice under Uncertainty: The Impact of Market Structure, University of Cologne, Working Paper Series in Economics, No. 23/2006.

Laffont, J.-J. and J. Tirole (1993). A Theory of Incentives in Procurement and Regulation, MIT Press.

Lagerlöf, Johan (2006). Equilibrium Uniqueness in a Cournot Model with Demand Uncertainty, *Topics in Theoretical Economics* 6, Article 19.

Littlechild, S. (1983). Regulation of British Telecommunications' profitability. Tech. rep., Department of Industry, London.

Pint, E. (1992). Price cap versus rate-of-return regulation in a stochastic cost model. *RAND Journal of Economics* 23(4), 564–578.

Stoft, S. (2002). Power Systems Economics - Designing Markets for Electricity. IEEE Press, John Wiley & Sons.

A Proof of Theorem 1

Denote by $\theta^Q(q) = \{\max\theta : P(Q,\theta) = z\}$ the demand scenario where the price rises above z (we then say that production is "binding") and by $\theta^{\bar{p}}(q,\bar{p}) = \{\theta : \bar{p} = P(Q,\theta)\}$ the demand realization where the price cap is met. Obviously it always holds that $\theta^Q(q) < \theta^{\bar{p}}(q,\bar{p})$, since the price cap can only be met if the price rises above z (i. e. production is already binding). The profit function is given by[35]

$$\pi_i(q,\bar{p}) = \int_{\underline{\theta}}^{\theta^Q(q)} zq_i dF(\theta) + \int_{\theta^Q(q)}^{\theta^{\bar{p}}(q,\bar{p})} P(Q,\theta)q_i dF(\theta) + \int_{\theta^{\bar{p}}(q,\bar{p})}^{\bar{\theta}} \bar{p}q_i dF(\theta) - cq_i. \tag{5}$$

Existence In order to prove existence we apply theorem 2.1 of Amir and Lambson (2000), p. 239. They show that the standard Cournot oligopoly game has at least one symmetric equilibrium and no asymmetric equilibria whenever demand $P(\cdot)$ is continuously differentiable and decreasing, cost $C(\cdot)$ is twice continuously differentiable and nondecreasing and, moreover, the cross partial derivative $\frac{d\pi_i(q,\bar{p})}{dQ_{-i}dQ} > 0$, where Q denotes total production and Q_{-i} production of the firms other than i. In order to see that the conditions required by Amir and Lambson are satisfied in our setup, note that in our game firms choose output given constant marginal cost and *expected* demand. Expected inverse demand is given by

$$EP(Q) = \int_{\underline{\theta}}^{\theta^Q(q)} zdF(\theta) + \int_{\theta^Q(q)}^{\theta^{\bar{p}}(q,\bar{p})} P(Q,\theta)\, dF(\theta) + \int_{\theta^{\bar{p}}(q,\bar{p})}^{\bar{\theta}} \bar{p}dF(\theta). \tag{6}$$

Note that $\frac{dEP(Q)}{dQ} = \int_{\theta^Q}^{\theta^{\bar{p}}} P_q(Q,\theta)dF(\theta) < 0$, and thus, $EP(Q)$ is strictly decreasing in Q. Moreover, the cross partial derivative[36]

$$\frac{d\pi^2(q,\bar{p})}{dQ_{-i}dQ} = -\int_{\theta^Q(q)}^{\theta^{\bar{p}}(q,\bar{p})} P_q(Q,\theta)dF(\theta) > 0$$

is positive. Thus, by Amir and Lambson (2000), theorem 2.1, there exists at least one symmetric equilibrium and no asymmetric equilibria.

Characterization Since no asymmetric equilibria exist in our framework, we now focus on the symmetric case and characterize equilibrium production. Denote the derivative of $\pi_i(q,\bar{p})$ (equation (5)) with respect to q_i by

$$\Gamma(q,\bar{p}) = \int_{\underline{\theta}}^{\theta^Q(q)} zdF(\theta) + \int_{\theta^Q(q)}^{\theta^{\bar{p}}(q,\bar{p})} \left[P_q(Q,\theta)q_i + P(Q,\theta)\right] dF(\theta) + \int_{\theta^{\bar{p}}(q,\bar{p})}^{\bar{\theta}} \bar{p}dF(\theta) - c, \tag{7}$$

[35]Note that for $\theta < \theta^Q(q)$ profits are zero if $z = 0$.

[36]See Amir and Lambson (2000), p. 238.

Note that $\frac{d\pi_i}{dq_i} > 0$ at $Q = 0$ (since production is assumed to be gainful), that $\frac{d\pi_i}{dq_i} < 0$ for some finite value of Q, and that $\frac{d\pi_i}{dq_i}$ is continuous. Thus, we have at least one point where (2) is satisfied and $\frac{d\pi_i}{dq_i}$ is decreasing. The characterization of equilibrium as given in theorem 1 follows straightforwardly (for easier exposition we have set z equal to zero in the theorem).

B Proof of Theorem 2

We first show that $\overline{MR} < \bar{\rho}^{\infty}$, i.e the interval $[\overline{MR}, \bar{\rho}^{\infty}]$ of improving price caps is always non–empty. To see this, denote by $\theta^{\overline{MR}} \leq \overline{\theta}$ the scenario where marginal revenue is maximized. Note that by assumption 1, part (ii), prices are strictly increasing in θ, and thus, it holds that $\bar{\rho}^{\infty} = P(Q^{*h}(\bar{\rho}^{\infty}), \overline{\theta}) > P(Q^{*h}(\bar{\rho}^{\infty}), \theta^{\overline{MR}})$. Moreover, note that $P(Q^{*h}(\bar{\rho}^{\infty}), \theta^{\overline{MR}}) > P(Q^{*h}(\bar{\rho}^{\infty}), \theta^{\overline{MR}}) + P_q(Q^{*h}(\bar{\rho}^{\infty}), \theta^{\overline{MR}})\frac{Q^{*h}(\bar{\rho}^{\infty})}{n} = MR(Q^{*h}(\bar{\rho}^{\infty}), \theta^{\overline{MR}})$, since $P_q(Q, \theta) < 0$ by assumption 1, part (i). Consequently, it holds that $\bar{\rho}^{\infty} = P(Q^{*h}(\bar{\rho}^{\infty}), \overline{\theta}) > MR(Q^{*h}(\bar{\rho}^{\infty}), \theta^{\overline{MR}}) = \overline{MR}$.

1) In order to prove that a price cap $\bar{p} \in [\overline{MR}, \bar{\rho}^{\infty}]$ increases production in the highest equilibrium, we pointwisely compare first order conditions for the respective values of Q.

Define $\widehat{Q} = \{Q : P(Q, \overline{\theta}) = c\}$ and

$$\Gamma(q, \bar{p}) = \int_{\underline{\theta}}^{\theta^{Q}(q)} z dF(\theta) + \int_{\theta^{Q}(q)}^{\theta^{\bar{p}}(q,\bar{p})} \left[P\left(Q, \theta\right) + P_q\left(Q, \theta\right) \frac{Q}{n}\right] dF\left(\theta\right) + \int_{\theta^{\bar{p}}(q,\bar{p})}^{\overline{\theta}} \bar{p} dF(\theta) - c. \tag{8}$$

Notice that $\Gamma(q, \bar{p})$ is just the first order condition as given in theorem 1. Given $\bar{p}$, Γ is a continuous function. Now note that for $Q \geq \widehat{Q}$ it holds that $\theta^{Q} \leq \theta^{\bar{p}} = \overline{\theta}$ and $P(Q, \theta) \leq c$, and thus, $\Gamma(\overline{q}, \bar{p}) < 0$ for all $\bar{p}$. Denote equilibrium production in absence of a price cap by $Q^{*\infty} := Q^{*h}(\bar{\rho}^{\infty})$. $Q^{*\infty}$ is the highest element solving

$$\Gamma(q, \bar{\rho}^{\infty}) \quad = \quad \int_{\underline{\theta}}^{\theta^{Q}(q^{*\infty})} z dF(\theta) + \int_{\theta^{Q}(q^{*\infty})}^{\overline{\theta}} \left[P\left(Q^{*\infty}, \theta\right) + P_q\left(Q^{*\infty}, \theta\right) \frac{Q^{*\infty}}{n}\right] dF\left(\theta\right) - c = 0.$$

Now introduce a price cap $\bar{p}^{0} \in [\overline{MR}, \bar{\rho}^{\infty}]$. It holds that

$$\begin{aligned} \Gamma(q^{*\infty}, \bar{p}^{0}) \quad = \quad & \int_{\underline{\theta}}^{\theta^{Q}(q^{*\infty})} z dF(\theta) + \int_{\theta^{Q}(q^{*\infty})}^{\theta^{\bar{p}}\left(q^{*\infty}, \bar{p}^{0}\right)} \left[P\left(Q^{*\infty}, \theta\right) + P_q\left(Q^{*\infty}, \theta\right) \frac{Q^{*\infty}}{n}\right] dF\left(\theta\right) \\ & + \int_{\theta^{\bar{p}}(q^{*\infty}, \bar{p}^{0})}^{\overline{\theta}} \bar{p}^{0} \, dF\left(\theta\right) - c \geq 0. \end{aligned}$$

To see this, note first that $\theta^Q(q^{*\infty})$ does not depend on $\bar{p}$. Moreover, note that with binding price cap $\bar{p} \in [\overline{MR}, \bar{p}^\infty]$ the integrand is pointwisely bigger than without binding price cap: if the price cap binds for some θ but before only production was binding, the integrand is increased. The reason is that the price cap is always higher than marginal revenue in any scenario (and marginal revenue is just the integrand in the first order condition without binding price cap).

Since $\Gamma(\cdot, \bar{p})$ is a continuous function and since $\Gamma(q^{*\infty}, \bar{p}^0) > 0$ and $\Gamma(\bar{q}, \bar{p}^0) < 0$, by the Mean Value Theorem there must exist at least one solution to $\Gamma(q, \bar{p}^0) = 0$ that satisfies $\Gamma_q(q, \bar{p}^0) < 0$ in the interval $]Q^{*\infty}, \widehat{Q}[$. Obviously, the highest of these solutions is $Q^{*h}(\bar{p}^0)$. It obviously holds that $Q^{*h}(\bar{p}^0) > Q^{*\infty}$.

Note that we cannot establish monotonicity results without further assumptions on the distribution $F(\theta)$, but for any distribution production in the highest equilibrium is higher in the presence of a binding price cap $\bar{p} \in [\overline{MR}, \bar{p}^\infty]$ than without a binding price cap.

2) Since production is always increasing for any price cap $\bar{p} \in [\overline{MR}, \bar{p}^\infty]$ (as compared to no price cap), consumer surplus also increases.

3) In order to show that also total welfare always increases if a price cap price cap $\bar{p} \in [\overline{MR}, \bar{p}^\infty]$ is chosen (as compared to no price cap) we first show the following

LEMMA 1 *Denote welfare optimal production given $\bar{p}$ by $Q^{**}(\bar{p})$. It always holds that $Q^*(\bar{p}) < Q^{**}(\bar{p})$.*

PROOF Welfare is given as follows,

$$W(Q) = \int_{\underline{\theta}}^{\bar{\theta}} \int_0^Q P(X, \theta) dX dF(\theta) - cQ. \tag{9}$$

Differentiation with respect to Q yields

$$\frac{dW(Q)}{dQ} = \int_{\underline{\theta}}^{\theta^Q(q)} z dF(\theta) + \int_{\theta^Q(q)}^{\bar{\theta}} P(Q, \theta) dF(\theta) - c, \tag{10}$$

and the welfare optimum is given by

$$Q^{**} = \left\{ Q : \int_{\underline{\theta}}^{\theta^Q(q)} z dF(\theta) + \int_{\theta^Q(q)}^{\bar{\theta}} P(Q, \theta) dF(\theta) = c \right\}, \tag{11}$$

where $\theta^Q(q)$ is the critical value of θ from which on production is binding.

Now consider the first order condition given in theorem 1 for a given price cap,

$$\frac{d\pi_i(q, \bar{p})}{dQ} = \int_{\underline{\theta}}^{\theta^Q(q)} z dF(\theta) + \int_{\theta^Q(q)}^{\theta^{\bar{p}}(q, \bar{p})} \left(P(Q, \theta) + P_q(Q, \theta) \frac{Q}{n} \right) dF(\theta) + \int_{\theta^{\bar{p}}(q, \bar{p})}^{\bar{\theta}} \bar{p} dF(\theta) - c, \tag{12}$$

In order to prove the lemma, we now show that for any fixed Q and for any $\overline{p}$, we obtain $\frac{dW(Q)}{dQ} > \frac{d\pi_i(q,\bar{p})}{dQ}$. We can get by pointwise inspection that for $\theta \in [\theta^Q, \theta^{\bar{p}}]$ we have $P > P + P_q \frac{Q}{n}$ and for $\theta \in [\theta^{\bar{p}}, \overline{\theta}]$ we have $P(Q,\theta) \geq \bar{p}$.

Whenever the distribution of uncertainty has strictly positive mass for two different values of θ, i.e. $f(\theta) > 0$ and $f(\theta') > 0$, where $\theta' \in]\tilde{\theta}^Q(q^{**}), \overline{\theta}[$, then it holds that $\frac{dW(\hat{Q})}{dQ} > \frac{d\pi_i(\hat{q},\bar{p})}{dQ}$ for all values of Q and $\bar{p}$. The above First order conditions will never coincide, since welfare is strictly concave. we can conclude that $Q^*(\bar{p}) < Q^{**}(\bar{p})$ for all $\bar{p}$. □

Now we have shown that production increases by setting a price cap $\bar{p} \in [\bar{\rho}^\infty, \bar{\rho}^\infty)$, but remains below the welfare optimal level. Since expected welfare as given by (9) is a concave function, expected total welfare with price cap obviously is higher than without price cap.

4) Finally we show that average prices are well defined and decrease in the relevant range. For a given $Q^*(\overline{p})$, average prices are uniquely given by

$$\begin{aligned}
\mathbf{E}[P] \;\; = \;\; & \int_{\underline{\theta}}^{\theta^Q(q^{*h}(\bar{p}))} z dF(\theta) \\
& + \int_{\theta^Q(q^{*h}(\bar{p}))}^{\theta^{\bar{p}}(q^{*h}(\bar{p}),\bar{p})} P(Q^{*h}(\overline{p}),\theta) dF(\theta) + \int_{\theta^{\bar{p}}(q^{*h}(\bar{p}),\bar{p})}^{\overline{\theta}} \overline{p} dF(\theta). \qquad (13)
\end{aligned}$$

If we lower pricecaps from $\bar{\rho}^\infty$ to some $\bar{p}^0 \in [\overline{MR}, \bar{\rho}^\infty]$, average prices are affected as follows. For the limits of integration in (13) it holds that $\theta^Q(q^{*h}(\bar{p}))$ and $\theta^p(q^{*h}(\bar{p}))$ increase. For the integrands it holds that $P(Q^{*\infty},\theta) > P(Q^{*h}(\bar{p}),\theta)$ and $\bar{\rho}^\infty > \bar{p}^0$. Thus, average prices decrease.

C Proof of Theorem 3

Preliminaries: Concavity of π_i for $\bar{p}$ close to $\bar{\rho}^0$. To prove the theorem we apply the Implicit Function Theorem in order to derive $\frac{dQ^*(\bar{p})}{d\bar{p}}$. To be able to do this, we need to make sure that $\frac{d^2\pi_i}{dqdQ} < 0$ holds for price caps close to $\bar{\rho}^0$ (which is either at marginal cost or above, see definition 1). We first show that for $\bar{p}$ close to $\bar{\rho}^0$ it always holds that $\theta^Q = \underline{\theta}$, and then, that this implies $\frac{d^2\pi_i}{dqdQ} < 0$.

First consider the case $Q^0 > 0$. Then, it holds that $\bar{\rho}^0 = c$, i.e. production is strictly positive (and equal to Q^0) as $\overline{p} \searrow c$. Then, for $\overline{p} > c$ and $\overline{p}$ close to c it holds that $Q^*(\overline{p})$ is close to Q^0. In fact, we can get arbitrarily close to Q^0 by choosing an appropriate $p > c$. Now define

$$\hat{\theta}^Q(q) = \{\theta : P(Q,\theta) = z\}$$

and note that

$$\theta^Q(q) = \max[\underline{\theta}, \hat{\theta}^Q(q)] \qquad (14)$$

Also recall that Q^0 is defined by the following equation,

$$P(Q^0, \underline{\theta}) = c. \tag{15}$$

Now note that it follows that $\hat{\theta}^Q(q^0) = \{\theta : P(Q^0, \theta) = z\} < \underline{\theta}$ for $\bar{p}$ close enough to c (recall that $c > z$). Thus, $\theta^Q(q^*(\bar{p})) = \underline{\theta}$, as asserted.

Now consider the case $Q^0 = 0$. Then, it holds that $P(0, \underline{\theta}) < c$ and $\hat{\theta}^Q = \{\theta : P(0, \theta) = z\}$. Thus, $\hat{\theta}^Q < \underline{\theta}$, which implies $\theta^Q = \underline{\theta}$.

Moreover, for $\bar{p} > c$ and $\bar{p}$ close to c, it always holds that $\theta^{\bar{p}} > \underline{\theta}$. To see this, consider the first order condition,

$$\int_{\underline{\theta}}^{\theta^{\bar{p}}} P(Q, \theta) + P_q(Q, \theta)\frac{Q}{n} dF(\theta) + \int_{\theta^{\bar{p}}}^{\bar{\theta}} \bar{p} dF(\theta) = c \tag{16}$$

For $\theta^{\bar{p}} = \underline{\theta}$ we would have $\int_{\underline{\theta}}^{\bar{\theta}} \bar{p} dF(\theta) > c$ for $\bar{p} > c$, and thus also for $\bar{p}$ close to c it must hold that $\theta^{\bar{p}} > \underline{\theta}$. Only in the limit, as $\bar{p} \searrow c$ we obtain $\theta^{\bar{p}} = \underline{\theta}$.

Consequently, for $\bar{p} > c$ and $\bar{p}$ close to c, it holds that the derivative $\frac{d^2\pi_i(q,\bar{p})}{dq_i dQ}$ is given by

$$\frac{d^2\pi_i(q, \bar{p})}{dq_i dQ} = \int_{\underline{\theta}}^{\theta^{\bar{p}}} \left[\left(1 + \frac{1}{n}\right) P_q(Q, \theta) + P_{qq}(Q, \theta)\frac{Q}{n}\right] dF(\theta) + \frac{d\theta^{\bar{p}}}{dQ} P_q(Q, \theta)\frac{Q}{n} f(\theta^{\bar{p}}) < 0$$

Thus, profits are concave for $\bar{p} > c$ and $\bar{p}$ close to c, and we can apply the Implicit Function Theorem in the neighborhood of $\bar{p}^0$.

We now derive $\frac{dQ^*(\bar{p})}{d\bar{p}}$ and then evaluate it the neighborhood of Q^0, as defined in definition 1 (which is the equilibrium production as $\bar{p} \searrow c$). We start from the equilibrium condition characterized in theorem 1, assuming that $\bar{p}$ is close to c such that it holds that $\theta^Q = \underline{\theta}$.

$$\frac{\partial \pi_i(q, \bar{p})}{\partial q_i} = \int_{\underline{\theta}}^{\theta^{\bar{p}}(q^*, \bar{p})} \left[P(Q^*, \theta) + P_q(Q^*, \theta)\frac{1}{n} Q^*\right] dF(\theta) + \int_{\theta^{\bar{p}}(q^*, \bar{p})}^{\bar{\theta}} \bar{p} dF(\theta) - c = 0.$$

Applying the Implicit Function Theorem yields[37]

$$\begin{aligned}
\frac{dQ^*(\bar{p})}{d\bar{p}} &= -\frac{\partial^2 \pi_i(q, \bar{p})}{\partial q_i \partial \bar{p}} \Big/ \frac{\partial^2 \pi_i(q, \bar{p})}{\partial q_i \partial Q} \\
&= \left[\frac{P_q(Q^*, \theta^{\bar{p}})}{P_\theta(Q^*, \theta^{\bar{p}})} \frac{Q^*(p)}{n} f(\tilde{\theta}^{\bar{p}}) + 1 - F(\theta^{\bar{p}})\right] \Big/ -\frac{\partial^2 \pi_i(q, \bar{p})}{\partial q_i \partial Q},
\end{aligned} \tag{17}$$

Notice that the denominator of (25) is always strictly positive since, as shown above, profits are strictly concave for $\bar{p}$ close to $\bar{p}^0$. Thus, it holds that

$$\frac{dQ^*(\bar{p})}{d\bar{p}} > (<) 0 \quad \Leftrightarrow \quad \frac{P_q(Q^*, \theta^{\bar{p}})}{P_\theta(Q^*, \theta^{\bar{p}})} \frac{Q^*(\bar{p})}{n} f(\theta^{\bar{p}}) + 1 - F(\theta^{\bar{p}}) > (<) 0$$

Note that as $\bar{p} \searrow c$, it holds that $Q^*(\bar{p}) \searrow Q^0$ and $\theta^{\bar{p}} \searrow \underline{\theta}$. Thus, in the limit, the following holds,

$$\left.\frac{dQ^*(\bar{p})}{d\bar{p}}\right|_{\bar{p} \searrow c} > (<) 0 \quad \Leftrightarrow \quad \frac{P_q(Q^0, \underline{\theta})}{P_\theta(Q^0, \underline{\theta})} \frac{Q^0}{n} f(\underline{\theta}) + 1 > (<) 0,$$

which yields the condition given in the theorem.

[37]Notice that $\frac{\partial \theta^{\bar{p}}}{\partial \bar{p}} = \frac{1}{P_\theta}$ follows immediately from partial differentiation of $P(Q, \theta^{\bar{p}}(q, \bar{p})) \equiv \bar{p}$.

D Proof of Theorem 4

Existence and Uniqueness In Grimm and Zoettl (2006) we show that the Cournot market game in absence of a price cap has a unique equilibrium which is symmetric (lemma 1). The assumptions under which the result can be shown are a bit more general than our assumptions 1 and 2 (in particular we allow for convex cost at both stages, production and sales[38]).

Now consider the following modified demand function that captures the situation that firms face a given price cap.

$$\bar{P}(Q,\theta) = \begin{cases} \bar{p} & \text{if} \quad 0 \leq Q \leq Q^{\bar{p}}(\theta) \\ P(Q,\theta) & \text{if} \quad Q > Q^{\bar{p}}(\theta), \end{cases} \tag{18}$$

where $Q^{\bar{p}}(\theta) = \{Q : P(Q,\theta) = \bar{p}\}$.

Note that $\bar{P}(Q,\theta)$ does not satisfy assumptions 1 and 2 and thus, the proof in Grimm and Zoettl (2006) does not directly apply to the case of a price cap. We have to deal with two issues:

(i) The fact that $\bar{P}_q(Q,\theta) = 0$ for $Q \in [0, Q^{\bar{p}}(\theta)]$. This implies that, given θ, the equilibrium at the second stage is not necessarily unique, which leads potentially to multiple equilibria of the overall game. Our assumption that for a binding price cap, sales are as equal as possible (see section 3.2) overcomes this problem. It can be interpreted either as (i) a special equilibrium selection[39], or (ii) as analyzing only a special class of equilibria of the overall game, focusing on symmetric outcomes at the second stage.

(ii) Furthermore demand $\bar{P}(Q,\theta)$ is kinked at $Q^{\bar{p}}(\theta)$. This implies that firms' stage two profits are not twice continuously differentiable. It seems obvious that this kink does not pose any problems when carrying through the proof along the lines of the proof of lemma 1 in Grimm and Zoettl (2006) since the kink is concave. Since the computations are mainly redundant to the proof conducted in Grimm and Zoettl (2006) (the only difference being the concave kink) they are not reported in detail here.

We conclude that the Cournot market game with free disposal has a unique equilibrium which is symmetric.

Characterization of Equilibrium Since from Grimm and Zoettl (2006) it follows that no asymmetric equilibria exist in our framework, we now focus on the symmetric case and characterize equilibrium production.

[38]The story behind the model in Grimm and Zoettl (2006) is that firms invest in capacity (denoted x in that paper) at the first stage and then produce at the second stage (production is denoted by q). Thus, the investment stage is equivalent to the production stage in this paper and the production stage is equivalent to the sales stage here. The notation has to be adapted accordingly.

[39]Note that this would be the unique equilibrium at the second stage if even for a binding price cap, inverse demand would have an (infinitely small) slope.

Let us first consider profits for each realization of θ, given q and $\bar{p}$. Recall that firms may sell strictly less than their production. Denote by $\hat{Y}(\theta)$ the unrestricted Cournot equilibrium quantity in scenario θ, which may be higher than Q. Denote by $Y^*(q,\theta)$ equilibrium sales in scenario θ, i. e. $Y^*(q,\theta) = \min\{\hat{Y}(\theta), Q\}$. Profits in scenario θ, given q and $\bar{p}$, are

$$\pi_i(q,\bar{p},\theta) = \begin{cases} P(Y^*(q,\theta),\theta)y_i^*(q,\theta) & \text{if} \quad \bar{p} \geq P(Y^*(q,\theta),\theta) \\ \bar{p}\frac{Y^{\bar{p}}(q,\theta)}{n} & \text{if} \quad \bar{p} < P(Y^*(q,\theta),\theta) \end{cases}, \tag{19}$$

where $Y^{\bar{p}}(q,\theta) = \min(Q, \{Y : \bar{p} = P(Y,\theta)\})$ Note that if production is binding it holds that $Y^*(q,\theta) = Q$, otherwise $Y^*(q,\theta) < Q$. If production is binding, two cases are possible: (i) the firms do not have an incentive to withhold, but the price cap is not yet met; (b) the price cap is effective and the firms sell their whole production[40]. Let us denote by $\tilde{\theta}^Q(q,\bar{p})$ the demand scenario from which on the firms sell their entire production and by $\tilde{\theta}^{\bar{p}}(q,\bar{p})$ the demand realization where firms sell their entire production *and* the price cap is met. $\tilde{\theta}^{\bar{p}}(q,\bar{p})$ and $\tilde{\theta}^Q(q,\bar{p})$ are implicitly defined as follows:

$$\begin{aligned} \tilde{\theta}^{\bar{p}}(q,\bar{p}) &= \{\theta : \bar{p} = P(Q,\theta)\} \\ \tilde{\theta}^Q(q,\bar{p}) &= \min\left(\tilde{\theta}^{\bar{p}}(q,\bar{p}), \{\theta : P_q(Q,\theta)q + P(Q,\theta) = 0\}\right) \end{aligned}$$

Since in $\theta \in [\tilde{\theta}^Q(q,\bar{p}), \overline{\theta}]$ the price is monotonically increasing in θ, profits in this interval are given by

$$\pi_i(q,\bar{p},\theta) = \begin{cases} P(Q,\theta)q_i & \text{for} \quad \theta \in [\tilde{\theta}^Q(q,\bar{p}), \tilde{\theta}^{\bar{p}}(q,\bar{p})] \\ \bar{p}q_i & \text{for} \quad \theta \in [\tilde{\theta}^{\bar{p}}(q,\bar{p}), \overline{\theta}] \end{cases}, \tag{20}$$

Denote $\pi_i(q,\bar{p},\theta)|_{\theta<\tilde{\theta}^Q(q,\bar{p})} = \pi_i^0(\bar{p},\theta)$. Now we are in the position to write down a firm's expected profits at stage one,

$$\pi_i(q,\bar{p}) = \int_{\underline{\theta}}^{\tilde{\theta}^Q(q,\bar{p})} \pi_i^0(\bar{p},\theta)dF(\theta) + \int_{\tilde{\theta}^Q(q,\bar{p})}^{\tilde{\theta}^{\bar{p}}(q,\bar{p})} P(Q,\theta)q_i dF(\theta) + \int_{\tilde{\theta}^{\bar{p}}(q,\bar{p})}^{\overline{\theta}} \bar{p}q_i dF(\theta) - cq_i.$$

Differentiation yields the first order condition (note that $\pi_i^0(\bar{p},\theta)$ does not depend on q_i and that the derivatives of the integration limits cancel out due to Leibnitz' rule)

$$\frac{d\pi_i(q,\bar{p})}{dq_i} = \int_{\tilde{\theta}^Q(q,\bar{p})}^{\tilde{\theta}^{\bar{p}}(q,\bar{p})} \left[P_q(Q,\theta)q_i + P(Q,\theta)\right] dF(\theta) + \int_{\tilde{\theta}^{\bar{p}}(q,\bar{p})}^{\overline{\theta}} \bar{p}dF(\theta) - c = 0. \tag{21}$$

From (21) the equilibrium characterization in Theorem 4 can be easily derived. If the price cap does not bind in any demand scenario, it holds that $\tilde{\theta}^{\bar{p}}(q,\bar{p}) = \overline{\theta}$ and the second

[40]Note that if at a certain θ the whole production is sold a the price cap, then the price cap is also binding in any higher demand scenario. This monotonicity does not hold, however, in demand scenarios where firms withhold quantity. Assumption 2, part (ii), guarantees that the Cournot quantity $\hat{Y}(\theta)$ is increasing in θ. It does not guarantee, however, that equilibrium *prices* are increasing in θ. Consequently, as θ increases, the price cap might be binding for low realizations and non–binding for higher ones.

integral on the LHS of the first order condition cancels out. If $\tilde{\theta}^{\bar{p}}(q, \bar{p}) < \overline{\theta}$, the price cap is binding in some demand scenarios. Two cases are possible: (1) production is binding before the price cap is met ($\tilde{\theta}^{Q}(q, \bar{p}) < \tilde{\theta}^{\bar{p}}(q, \bar{p})$), or production and price cap become binding in the same demand scenario ($\tilde{\theta}^{Q}(q, \bar{p}) = \tilde{\theta}^{\bar{p}}(q, \bar{p})$). The latter case occurs whenever marginal revenue in scenario $\tilde{\theta}^{\bar{p}}(q, \bar{p})$ is negative.

E Proof of Theorem 5

For high price caps $\bar{p} \in [\overline{MR}, \bar{\rho}^{\infty}), \bar{\rho}^{\infty}]$ the proof goes along the same lines as the proof of theorem 2.

For low price caps $\bar{p} \searrow c$, the result is proven along the same lines as theorem 3. The main difference is that under free disposal the equilibrium is unique and expected profits are strictly concave for all $\bar{p}$. To see this, we derive

$$\frac{d^2\pi_i(q, \bar{p}; y^*)}{dq_i dQ} = \int_{\tilde{\theta}^Q}^{\tilde{\theta}^{p}} \left(P_q(Q, \theta) \left(1 + \frac{1}{n}\right) + P_{qq}(Q, \theta)\frac{Q}{n} \right) dF(\theta) + \frac{d\tilde{\theta}^{\bar{p}}}{dQ} P_q(Q^*, \tilde{\theta}^{\bar{p}}) \frac{Q^*}{n} f(\tilde{\theta}^{\bar{p}})$$

Note that the first term is strictly negative (by assumption 1, part (ii)) whenever it holds that $\tilde{\theta}^{Q} < \tilde{\theta}^{\bar{p}}$, and zero otherwise. The second term is always strictly negative since θ has full support.

F Proof of Theorem 6

The Competitive Equilibrium Denote by Q^*_{PC} equilibrium investment under perfect competition in absence of a price cap. The proof of existence and uniqueness follows the same lines as the proof in case of oligopoly (theorem 4). Note that under perfect competition firms do not strategically withhold produced quantity. Thus, the price rises above z only if $\theta > \theta^Q$, as defined in the proof of theorem 1 (note that we do not need a critical value of θ where the price cap is met since we only consider the case of non–binding price caps). The profit function under perfect competition is given by[41]

$$\pi_i(q, \bar{p}) = \int_{\theta^Q(q)}^{\overline{\theta}} P(Q, \theta) q_i dF(\theta) - cq_i.$$

Thus, the first order condition of a perfectly competitive firm (that cannot affect the market price by changing its quantity) is given by

$$\int_{\theta^Q(q)}^{\overline{\theta}} P(Q, \theta) dF(\theta) = c. \tag{22}$$

[41]Note that for $\theta < \theta^Q(q, \bar{p})$ profits are zero.

Equivalence of p^{PC} and $\overline{MR}$ Now assume that demand can be decomposed such that $P(Q,\theta) = a(\theta) + b(\theta)\widetilde{P}(Q)$ where $a(\theta)$, $b(\theta)$, and $\widetilde{P}(Q)$ are such that assumptions 1 and 2 are satisfied. We show that, in absence of a price cap, the perfectly competitive equilibrium price just equals marginal equilibrium revenue in a market with a finite number of firms, i. e.

$$P(Q^*_{PC},\theta) = P(Q^{*\infty},\theta) + P_q(Q^{*\infty},\theta)\frac{Q^{*\infty}}{n} \quad \forall \quad \theta \in [\tilde{\theta}^Q(q^{*\infty}) = \theta^Q(q^{*\infty}), \overline{\theta}],$$

Thus, setting the highest price observed in the perfectly competitive benchmark is equivalent to setting the lowest price from the interval of welfare increasing price caps identified in theorem 2.

We now show that in absence of a price cap the equilibria of the market game under perfect competition and the Cournot market game with free disposal can be characterized by the same condition. To this end, define $T(Q^*_{PC}) := \widetilde{P}(Q^*_{PC})$ and $T(Q^{*\infty}) := \widetilde{P}(Q^{*\infty}) + \widetilde{P}_q(Q^{*\infty})\frac{Q^{*\infty}}{n}$ (recall that Q^*_{PC} denotes total equilibrium quantity under perfect competition, while $Q^{*\infty}$ denotes the Cournot outcome). Observe that the unique equilibrium production in both cases is characterized by the following equation,

$$\int_{\tilde{\theta}^T}^{\overline{\theta}} (a(\theta) + b(\theta)T)\, dF(\theta) = c, \tag{23}$$

where $\tilde{\theta}^T = \{\theta : a(\theta) + b(\theta)T = 0\}$.

Thus both, the solution under perfect competition and under imperfect competition involve an identical value T^* solving (23). For all $\theta \in [\theta^{T^*}, \overline{\theta}]$ we obtain

$$a(\theta) + b(\theta)T^* = P(Q^*_{PC},\theta) = P(Q^{*\infty},\theta) + P_q(Q^{*\infty},\theta)\frac{Q^{*\infty}}{n}.$$

G Proof of Theorem 7

We prove theorem 7 by setting $\frac{dQ^*(\bar{p})}{d\bar{p}}$ equal to zero. We then show that for an increasing hazard rate and $P_{q\theta} = 0$, $Q^*(\bar{p})$ is quasiconcave by proving $\frac{d^2Q^*(\bar{p})}{d\bar{p}^2}\Big|_{\frac{dQ^*(\bar{p})}{d\bar{p}}=0} < 0$.

Preliminaries As a first step we derive properties and slopes of $\tilde{\theta}^{\bar{p}}(q,\bar{p})$. Notice that at the demand realization where the price cap is met in equilibrium it holds that

$$P(Q, \tilde{\theta}^{\bar{p}}(q,\bar{p})) \equiv \bar{p}$$

We first calculate the partial derivative by differentiation with respect to $\bar{p}$ given some fixed Q,[42]

$$\frac{\partial\tilde{\theta}^{\bar{p}}(q,\bar{p})}{\partial\bar{p}} = \frac{1}{P_\theta(Q,\tilde{\theta}^{\bar{p}})} = 1.$$

[42]Note that $P_\theta = 1$ under our assumptions.

Now we derive the total derivative at $Q^*(\bar{p})$,

$$P_q(\cdot)\frac{dQ^*(\bar{p})}{d\bar{p}} + P_\theta(\cdot)\frac{d\tilde{\theta}^{\bar{p}}(q^*(\bar{p}),\bar{p})}{d\bar{p}} = 1 \quad \Leftrightarrow \quad \frac{d\tilde{\theta}^{\bar{p}}(q^*(\bar{p}),\bar{p})}{d\bar{p}} = 1 - P_q(Q^*,\theta)\frac{dQ^*(\bar{p})}{d\bar{p}}. \tag{24}$$

Characterization of $\bar{p}^*$ Now recall from the proof of theorem 5 that under free disposal expected profits are strictly concave for all $\bar{p}$. Thus, we can apply the Implicit Function Theorem in order to derive $\frac{dQ^*(\bar{p})}{\bar{p}}$. Differentiation of the first order condition

$$\int_{\tilde{\theta}^Q(q^*,\bar{p})}^{\tilde{\theta}^{\bar{p}}(q^*,\bar{p})} \left[P\left(Q^*,\theta\right) + P_q\left(Q^*,\theta\right)\frac{1}{n}Q^*\right] dF\left(\theta\right) + \int_{\tilde{\theta}^{\bar{p}}(q^*,\bar{p})}^{\bar{\theta}} \bar{p}dF\left(\theta\right) - c = 0.$$

with respect to $\bar{p}$ yields

$$\begin{aligned}\frac{dQ^*(\bar{p})}{d\bar{p}} &= -\frac{\partial^2\pi_i(q,\bar{p})}{\partial q_i\partial\bar{p}} \Big/ \frac{\partial^2\pi_i(q,\bar{p})}{\partial q_i\partial Q} \\ &= \left[\frac{P_q(Q^*,\tilde{\theta}^{\bar{p}})}{P_\theta(Q^*,\tilde{\theta}^{\bar{p}})}\frac{Q^*(p)}{n}f(\tilde{\theta}^{\bar{p}}) + 1 - F(\tilde{\theta}^{\bar{p}})\right] \Big/ -\frac{\partial^2\pi_i(q,\bar{p})}{\partial q_i\partial Q},\end{aligned} \tag{25}$$

Notice that it may happen that $\tilde{\theta}^{\bar{p}}\left(q^*,\bar{p}\right) = \tilde{\theta}^Q\left(q^*,\bar{p}\right)$, in which case it holds that

$$-P_q(Q^*,\tilde{\theta}^{\bar{p}})\frac{Q^*}{n} = -P_q(Q^*,\tilde{\theta}^Q)\frac{Q^*}{n} = P(Q^*,\tilde{\theta}^Q) = P(Q^*,\tilde{\theta}^{\bar{p}}) = \bar{p}$$

(we denote this type of equilibrium by EQ^{II}, and the other type by EQ^I in the following). Rewriting equation (25) using the hazard rate yields

$$EQ^I: \quad \frac{dQ^*(\bar{p})}{d\bar{p}} = \left[P_q(\cdot)\frac{Q^*(p)}{n} + \frac{1}{h(\tilde{\theta}^{\bar{p}})}\right] \Big/ \left[-\frac{\partial\pi_i(q,y^*)}{\partial q_i\partial Q}\frac{1}{f(\tilde{\theta}^{\bar{p}})}\right] \tag{26}$$

$$EQ^{II}: \quad \frac{dQ^*(\bar{p})}{d\bar{p}} = \left[-\bar{p} + \frac{1}{h(\tilde{\theta}^{\bar{p}})}\right] \Big/ \left[-\frac{\partial\pi_i(q,y^*)}{\partial q_i\partial Q}\frac{1}{f(\tilde{\theta}^{\bar{p}})}\right] \tag{27}$$

Define $\xi(\bar{p}) = \left[-\frac{\partial\pi_i(q,y^*)}{\partial q_i\partial Q}\frac{1}{f(\tilde{\theta}^{\bar{p}})}\right]$. Note that it holds that $\xi(\bar{p}) > 0$ since profits (along the symmetry line) are concave. Thus, $\frac{dQ^*(\bar{p})}{d\bar{p}} = 0$ gives exactly the characterization of the production maximizing price caps as stated in the theorem.

In order to show existence and uniqueness of these maximizers, however, we also need to show that second order conditions are satisfied, namely $\left.\frac{d^2Q^*(\bar{p})}{d\bar{p}^2}\right|_{\frac{dQ^*(\bar{p})}{d\bar{p}}=0} < 0$. Differentiation of (26) and (27) with respect to $\bar{p}$ yields[43]

$$EQ^I: \quad \frac{d^2Q^*(\bar{p})}{d\bar{p}^2} = \frac{1}{\xi(\bar{p})^2}\left[\left(\frac{dQ^*(\bar{p})}{d\bar{p}}\left(P_{qq}\frac{Q^*(\bar{p})}{n} + P_q\right) - \frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})}\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}}\right)\xi(\bar{p}) - \frac{dQ^*(\bar{p})}{d\bar{p}}\xi(\bar{p})\xi_{\bar{p}}(\bar{p})\right]$$

$$EQ^{II}: \quad \frac{d^2Q^*(\bar{p})}{d\bar{p}^2} = \frac{1}{\xi(\bar{p})^2}\left[-\left(1 + \frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})}\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}}\right)\xi(\bar{p}) - \frac{dQ^*(\bar{p})}{d\bar{p}}\xi(\bar{p})\xi_{\bar{p}}(\bar{p})\right]$$

[43] Notice that $P_{q\theta} = P_{\theta\theta} = 0$ and that $\frac{d\tilde{\theta}^{\bar{p}}}{d\bar{p}} = 1$ at $\frac{dQ^*(\bar{p})}{d\bar{p}} = 0$. Moreover, $\frac{dQ^*(\bar{p})}{d\bar{p}}\xi_i(\bar{p})$ is just the enumerator of expressions (26) and (27).

We get

$$EQ^I: \quad \left.\frac{d^2Q^*(\bar{p})}{d\bar{p}^2}\right|_{\frac{dQ^*(\bar{p})}{d\bar{p}}=0} = -\frac{1}{\xi(\bar{p})}\frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})} < 0$$

$$EQ^{II}: \quad \left.\frac{d^2Q^*(\bar{p})}{d\bar{p}^2}\right|_{\frac{dQ^*(\bar{p})}{d\bar{p}}=0} = -\frac{1}{\xi(\bar{p})}\left(1+\frac{h_\theta(\tilde{\theta}^{\bar{p}})}{h^2(\tilde{\theta}^{\bar{p}})}\right) < 0$$

Thus, under our assumptions all (local) extrema are necessarily maxima. Since we have a local minimum at $p = \bar{\rho}^\infty$, we conclude that either our first order condition characterizes the unique production maximizing price cap if $\frac{dQ^*(\bar{p})}{d\bar{p}}|_{\bar{p}=c} > 0$, otherwise $\bar{p} = c$ is optimal.