Strategic Capacity Choice under Uncertainty: The Impact of Market Structure on Investment and Welfare*

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Abstract

We analyze a market game where firms choose capacities under uncertainty about future market conditions and make output choices after uncertainty has unraveled. We show existence and uniqueness of equilibrium under imperfect competition and establish that capacity choices by strategic firms are generally too low from a welfare point of view. We also demonstrate that strategic firms choose even lower capacities if they anticipate competitive spot market pricing (e.g. due to regulatory intervention). We finally illustrate how the model can be used to assess the impact of electricity market liberalization on total capacity and welfare by fitting it to the data of the German electricity market.

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1 Introduction

In this article we investigate the nature of equilibrium outcomes in oligopolistic markets where firms make capacity choices under uncertainty about future market conditions (and/or anticipating fluctuating demand) and decide on output after the state of nature has unraveled. The fact that in many industries where non storable goods are produced, capacity is a long run decision, whereas production may be adjusted short–run is a natural motivation for our approach. Consider, for example, the electricity sector or the High Tech industry, where production has to take place just in time, but capacities have to be installed well in advance. In those markets firms usually face considerable demand and cost uncertainty when choosing their capacities. This may be due to uncertainty about the economic trend, about the success of a new product, about future weather conditions, or fuel prices, to give just a few examples. In electricity markets it is moreover well known that demand fluctuates systematically over each day, month, or year. Firms naturally anticipate those patterns when they make their investment decisions.

For a competitive industry investment incentives prior to spot market competition have been analyzed by the peak load pricing literature (see e.g. Crew and Kleindorfer (1986) for an overview).[1] It is shown that under perfect competition spot prices and quantities are determined by the intersection of (short run) marginal cost and demand, and that investment incentives are such that the social optimum is attained. It is important to note that in this "competitive benchmark" spot market prices may rise considerably above marginal cost of the *last* unit produced in case the capacity bound is reached. The reason is that the marginal cost curve is vertical at the capacity bound, and therefore prices are driven by the demand side in case the capacity constraint is binding.

Little is known, however, about the investment incentives of strategic firms under uncertain or fluctuating demand. Gabszewicz and Poddar (1997) were the first to analyze capacity choice by strategic firms prior to Cournot competition. They demonstrate in a linear duopoly model existence of a symmetric equilibrium. Reynolds and Wilson (2000) show that a two stage game where firms first invest under demand uncertainty and then engage in Bertrand competition after uncertainty unraveled has no symmetric pure strategy equilibrium.[2] Fabra and de Frutos (2006) take up on this and characterize asymmetric

[1]There is also an extensive literature that analyzes the impact of demand uncertainty on expected profits for monopolistic and competitive industries. See, for example, Oi (1961), Samdmo (1971), Leland (1972), or Dréze and Gabszewicz (1967).

[2]Kreps and Scheinkman (1983) and Osborne and Pitchik (1986) have shown that for certain demand

equilibria of this game. Garcia and Stacchetti (2007) analyze Markovian equilibria of a strategic dynamic model with random demand growth and find that firms have incentives to hold low reserve margins in order to extract higher rents.

In our paper we take up on the analysis of capacity choices under uncertainty prior to Cournot competition. This framework allows to answer the question: How does market power in the short run (at the production stage) affect investment incentives in the long run? This question obviously cannot be answered in a model where firms engage in Bertrand competition at stage two, since in that model unconstrained firms do not exercise market power. We believe that such a model would not describe the markets we are interested in.[3] Our analysis provides existence and uniqueness results in a general oligopoly model with endogenous investment that allows for either cost or demand uncertainty. This allows to answer some of the key questions concerning the investment incentives of strategic firms: First, we show that total capacity installed by strategic firms is too low both, locally (that is, given that firms behave strategically at the production stage) as well as globally (as compared to the first best competitive benchmark as analyzed by the peak load pricing literature). Obviously, strategic withholding is practiced not only at the production stage but also at the investment stage. Second, we analyze investment incentives in case strategic firms anticipate a competitive spot market outcome (for example since they expect regulatory intervention at the spot market). We show that in this case investment is lower than in the case where firms anticipate the Cournot outcome at stage two. Furthermore, uniqueness of equilibrium cannot be established. The result demonstrates that intervention only at the spot market may have undesirable effects if investment is endogenous. The welfare effect of spot market regulation is ambiguous in our model with endogenous capacity choice and may go in either direction.

We finally demonstrate how our theoretical insights can be used to assess long run effects of electricity market liberalization on capacity levels and to quantify the capacity and welfare effects of several recent policy proposals. We conduct our empirical analysis based on data of the German electricity market. We use estimates of short run demand elasticity, as well as data on variable production cost and investment cost in order to compute total

the Cournot outcome obtains if firms choose capacities prior to price competition.

[3] Having in mind applications like electricity markets, one could also opt for auctions or supply function competition to model competition in the short run. However, those models typically have multiple equilibria which limits the tractability of a model with endogenous investment decisions. Moreover, the Cournot outcome seems to be a good approximation of coalition–proof supply function equilibria, as Delgado and Moreno (2004) show.

capacity predicted by our model for the scenarios we analyze, for different degrees of market concentration. Comparison with currently installed capacity yields that actual capacity is rather close to the First Best level. This is presumably due to the regulatory regime in the pre–liberalization period which imposed too high investment incentives. In accordance with this observation we find that observed prices during the hours where capacity is binding (approximately 12 % of the year) are rather close to the first best level. Our theoretical model, however, implies that those prices are not sufficient to sustain the actual capacity level if investment is chosen strategically. In the long run, we detect a high potential for the exercise of market power through capacity withholding, which would significantly raise average prices well above the current level. We finally quantify the welfare effect of tight market monitoring at the production stage, and its direction. It turns out to be negative in concentrated industries, while it is slightly positive for more competitive markets.[4] Our empirical study demonstrates that our model adds important aspects to the ongoing debate on market power in electricity markets, which often ignores the possibility of strategic investment and focuses solely at assessment of the short run behavior of firms.[5]

Our paper is organized as follows: In section 2 we state the model. Section 3 contains the theoretical analysis and results. We consider strategic investment in section 3.1 and welfare optimal investment in section 3.2. In section 3.3 we provide a comparison of investment levels in the scenarios we consider and explore in more detail under what conditions the presence of uncertainty crucially affects the conclusions. Section 4 contains the empirical analysis, where we also discuss the welfare implications of spot market regulation. Section 5 concludes.

2 The Model

We analyze a two stage market game where firms have to choose capacities under demand and cost uncertainty, and make output choices after market conditions unraveled. We denote by $q = (q_1, \ldots, q_n)$ the vector of outputs of the n firms, and by $Q = \sum_{i=1}^{n} q_i$ total quantity produced in the market.

Inverse Demand is given by the function $P(Q, \theta)$, which depends on $Q \in \mathbb{R}^+$, and the random variable $\theta \in \mathbb{R}$ which represents uncertainty. Moreover, all firms face the same

[4]This is also confirmed by a simplified theoretical analysis with linear demand and uniform distribution of uncertainty.

[5]See, e.g. Schwarz and Lang (2006) for Germany, Joskow and Kahn (2002) for California, or Wolfram (1999) for the United Kingdom.

cost function for each $\theta \in \mathbb{R}$, which we denote by $C(q_i, \theta)$. The random variable $\theta \in \mathbb{R}$ is distributed according to a distribution $F(\theta)$ with bounded support.[6]

We introduce the parameter $z \leq 0$ as a lower bound on market prices in order to take into account nonnegativity of prices ($z = 0$) or disposal cost ($z < 0$). We denote the quantity where this lower bound is met by $\overline{Q}(\theta)$.[7] The following two assumptions on demand and cost for each realization of uncertainty $\theta \in \mathbb{R}$ have to be satisfied only for quantities $0 \leq q_i \leq Q < \overline{Q}(\theta)$.

Assumption 1 (Assumptions at each θ) *(i) Inverse demand $P(Q, \theta)$ is twice continuously differentiable*[8] *in Q with $P_q(Q, \theta) < 0$ and $P_q(Q, \theta) + P_{qq}(Q, \theta)q_i < 0$.*

(ii) $C(q_i, \theta)$ is twice continuously differentiable in q_i with $C_q(q_i, \theta) \geq 0$ and $C_{qq}(q_i, \theta) \geq 0$.

Assumption 2 (Monotonicity Assumptions regarding θ) *(i) $P(Q, \theta)$ and $C(q_i, \theta)$ are differentiable in θ, and it holds that $P_\theta(Q, \theta) - C_{q\theta}(q_i, \theta) > 0$.*[9]

(ii) $P(Q, \theta)q_i - C(q_i, \theta)$ is (differentiable) strict supermodular in q_i and θ, i. e. $P_\theta(Q, \theta) - C_{q\theta}(q_i) + P_{q\theta}(Q, \theta)q_i > 0$.

The situation we want to analyze is captured by the following two stage game. At stage one firms simultaneously build up capacities $x = (x_1, \ldots, x_n)$. Capacity choices are observed by all firms. Cost of investment $K(x_i)$ is the same for all firms and satisfies

[6] While F has bounded support, it will be convenient to assume that $P(Q, \theta)$ is defined for all $\theta \in \mathbb{R}$ and $Q \in \mathbb{R}_+$.

[7] In case the lower bound is not binding we can set $\overline{Q}(\theta) = \infty$. In order to ensure a bounded solution we then have to assume $\lim_{Q\to\infty} P(Q, \theta) < C_q(0, \theta)$ for each $\theta \in (-\infty, \infty]$.

[8] Throughout the paper we denote the derivative of a function $g(x, y)$ with respect to the argument x, by $g_x(x, y)$, the second derivative with respect to that argument by $g_{xx}(x, y)$, and the cross derivative by $g_{xy}(x, y)$.

[9] Notice that demand and cost uncertainty in principle can be driven by separate random events. Then the parameter θ denotes then all joint realizations of those events, which have to satisfy assumption 2. This requirement imposes some further restrictions on the model if cost and demand uncertainty should be considered simultaneously. Consider, for example, a model with linear demand $P(Q, \beta) = \beta - bQ$ and fluctuating but constant marginal cost $c(\gamma)$. For ease of exposition let both, β and γ follow a discrete distribution. Now sort all joint realizations (β, γ) such that $\beta - c(\gamma)$ is increasing and index each realization by θ. Observe that the resulting system satisfies assumption 2 (i) and 2 (ii). Thus, the model can deal simultaneously with cost and demand uncertainty in the case of linear demand, which we exploit in the empirical part of the paper. In case of non–linear demand it is more plausible to think about demand and cost uncertainty separately.

Assumption 3 (Investment Cost) *Investment cost $K(x_i)$ is twice continuously differentiable, with $K_x(x_i) \geq 0$ and $K_{xx}(x_i) \geq 0$.*

At stage two, facing the capacity constraints inherited from stage one, firms simultaneously choose outputs at the spot market. Since demand uncertainty unravels prior to the output decision, produced quantities depend on the realized demand scenario. We denote individual quantities produced in demand scenario θ by $q(\theta) = (q_1(\theta), \ldots, q_n(\theta))$, and the aggregate quantity by $Q(\theta) = \sum_{i=1}^{n} q_i(\theta)$.

Finally, we state firm i's stage one expected profit from operating if capacities are given by x and firms plan to choose feasible[10] production schedules $q(\theta)$ for all $\theta \in [-\infty, \infty]$.

$$\pi_i(x, q) = \int_{-\infty}^{\infty} \left[P\left(Q\left(\theta\right), \theta\right) q_i\left(\theta\right) - C\left(q_i\left(\theta\right), \theta\right)\right] dF\left(\theta\right) - K\left(x_i\right). \tag{1}$$

Throughout the paper we consider only cases where investment is gainful, i.e. $K(0) < \mathbf{E}_\theta[P(0,\theta) - C(0,\theta)]$. Note that if the condition does not hold, no firm invests in capacity.

3 Results

In this section we analyze the two stage market game where at stage one firms simultaneously invest in capacity under uncertainty about future market conditions and at stage two, when uncertainty has unraveled, decide on production. In order to be able to assess the impact of market power and of regulatory interventions on investment incentives and production, we analyze four different scenarios.

In section 3.1 we consider the case that strategic firms choose profit maximizing investment levels. In this context we consider both, the case of Cournot competition at the spot market as well as the case of competitive pricing (which may be a result of regulatory intervention). We are aware that the latter scenario requires a lot of information on the part of the social planer. Although stylized, however, it allows detailed insights in what happens to investment incentives should the regulator suceed in implementing competitive prices at the spot market.

In section 3.2 we assume that socially optimal investment levels are chosen at stage one (e.g. enforced by a social planer) and again consider Cournot competition as well as the case of competitive pricing at the spot market. The latter scenario coincides with the competitive benchmark that has been analyzed by the peak load pricing literature. Table 2 relates the four scenarios we consider.

[10]That is, $0 \leq q_i(\theta) \leq x_i$ for all $\theta \in [-\infty, \infty]$, $i = 1, \ldots, n$.

		Objective at the Production Stage	
		Profit (strategic firms)	**Welfare**
Objective at the Investment Stage	**Profit** (str. firms)	**Cournot** (total investment X^C)	**Marginal Cost Pricing** (total investment X^{MC})
	Welfare	**Second Best** (total investment X^{SB})	**First Best** (total investment X^{FB})

Table 1: The four scenarios analyzed.

3.1 Strategic Investment

First consider the market game where firms strategically choose capacities at stage one as to maximize profits. Our first theorem shows that the two stage market game where firms engage in Cournot competition (C) at stage two has a unique and symmetric equilibrium. If, however, a social planer intervenes at the production stage (implementing marginal cost (MC) pricing whenever firms are unconstrained), uniqueness can no longer be established and also existence cannot be guaranteed for general production costs.

Theorem 1 (Strategic Capacity Choice) *Suppose strategic firms choose their capacities at stage one.*

(C) If firms engage in Cournot competition at the second stage, the game has a unique equilibrium which is symmetric.

(MC) Suppose that firms anticipate marginal cost pricing at stage two, and that $C_q(q,\theta)$ is constant in q. Then, there exists at least one symmetric equilibrium, but there may be more than one. No asymmetric equilibria exist.

Total equilibrium investment in scenario $S \in \{C, MC\}$, X^S, solves

$$\int_{\tilde{\theta}^S(X^S)}^{\infty} \left[P\left(X^S,\theta\right) + P_q\left(X^S,\theta\right)\frac{X^S}{n} - C_q\left(\frac{X^S}{n},\theta\right)\right] dF\left(\theta\right) = K_x\left(\frac{X^S}{n}\right),$$

where $\tilde{\theta}^S\left(X^S\right)$ is the demand scenario from which on firms are capacity constrained at stage two.[11]

[11] I.e. θ^C is implicitly defined by $P(X^C,\tilde{\theta}^C)+P_q(X^C,\tilde{\theta}^C)\frac{X^C}{n} = C_q(\frac{X^C}{n},\tilde{\theta}^C)$ and θ^{MC} is implicitly defined by $P(X^{MC},\tilde{\theta}^{MC}) = C_q(\frac{X^{MC}}{n},\tilde{\theta}^{MC})$, respectively.

PROOF See appendix B □

Let us emphasize some important aspects of our results. First, we could show that *under standard regularity assumptions* the Cournot market game (i.e. the game where firms act strategically at both stages) has a *unique* equilibrium. Second, we find that (symmetric) equilibrium investment can be characterized by a rather intuitive condition. The condition simply says that expected marginal profit generated by an additional unit of capacity at the second stage must equal marginal cost of investment. When calculating the marginal profit generated by an additional unit of capacity, however, one has to take into account that additional capacity affects a firm's profit only in those states of nature where capacity is binding. Thus, expectation must only be taken with respect to those scenarios in which the firms are capacity constrained, i. e. over the interval $[\tilde{\theta}\left(X^S\right), \infty]$, and not over the whole domain of θ.

Note that the critical demand scenario $\tilde{\theta}$ (from which on firms are capacity constrained) depends on the market game at stage two. If firms strategically withhold production at the spot market (as under Cournot competition) the critical demand scenario is higher than in the case where they are forced to the competitive production schedule. Observe that actually the market game at stage two enters into the first order condition solely through the critical demand realization.

If firms anticipate that at stage two the welfare optimum given their capacity choices is implemented, existence and uniqueness of a symmetric equilibrium cannot be shown in the general case (part (MC) of the theorem). Only for constant marginal production cost we obtain existence (but not uniqueness).[12] An immediate insight of this result is that intervention at stage two may lead to high strategic uncertainty for the firms. Later in section 3.3 we will show that intervention at stage two moreover decreases investment incentives.

3.2 Optimal Investment

In order to assess the impact of market power at stage one on investment incentives, we now characterize welfare optimal capacity levels. Again, we consider both, the case of Cournot

[12]The basic problem is that in neither case the stage one profit is quasiconcave, which makes standard analysis impossible. In the case of linear marginal cost, however, we can exploit recent insights on oligopolistic competition that makes use of lattice theory (Amir (1996) and Amir and Lambson (2000)). In the general case (i. e. strictly convex production cost), however, the game cannot be reformulated as a supermodular game and thus, even more sophisticated techniques do not help.

competition at stage two (Second Best solution — SB), as well as the case of welfare optimal production (First Best solution — FB).

THEOREM 2 (WELFARE MAXIMIZATION AT STAGE ONE) *Suppose capacities are chosen at stage one as to maximize social welfare.*

(SB) If firms engage in Cournot competition at the second stage, welfare maximizing capacities are unique and symmetric.

(FB) If either production or investment costs are strictly convex, the First Best solution (that maximizes total welfare) is unique and symmetric.

Socially optimal investment in scenario $W \in \{SB, FB\}$, X^W, solves

$$\int_{\tilde{\theta}^W(X^W)}^{\infty} \left[P\left(X^W, \theta\right) - C_q\left(\frac{1}{n}X^W, \theta\right)\right] dF(\theta) = K_x\left(\frac{1}{n}X^W\right), \tag{2}$$

where $\tilde{\theta}^W\left(x^W\right)$ is the demand scenario from which on firms are capacity constrained at stage two.[13]

PROOF See appendix C □

Note that also the characterization of welfare optimal investment levels is rather intuitive. The condition implies that in the welfare optimum capacity should be chosen such that expected marginal social welfare generated by an additional unit of capacity [LHS of (2)] should equal marginal cost of investment [RHS of (2)]. Again it is important to notice that expectation is only taken over those scenarios where the firms are actually constrained *given the scheduled stage two–production*, that is, over the interval $[\tilde{\theta}^S(X^S), \infty]$. Note that for a given investment, firms are constrained earlier if socially optimal production is implemented at stage two since under Cournot competition they withhold quantity in order to affect prices. Consequently, additional capacity is used more often (or, with a higher probability) and thus, contributes more to expected marginal welfare if the spot market behavior is more competitive. This implies that the First Best capacity level should be higher than the Second Best. We show this formally in section 3.3.

We finally point out that if firms do not act strategically, investment and production levels coincide with the first best (socially optimal) solution, again given the number of firms:

[13]I.e. θ^{SB} is implicitly defined by $P(X^{SB}, \tilde{\theta}^{SB}) + P_q(X^{SB}, \tilde{\theta}^{SB})\frac{X^{SB}}{n} = C_q(\frac{X^{SB}}{n}, \tilde{\theta}^{SB})$ and θ^{FB} is implicitly defined by $P(X^{FB}, \tilde{\theta}^{FB}) = C_q(\frac{X^{FB}}{n}, \tilde{\theta}^{FB})$, respectively.

Remark 1 (Non-Strategic Firms) *For each number of firms, n, if firms do not behave strategically (i. e. they act as price takers at stage two and ignore their impact on total capacity at stage one), firms invest and produce optimally from a social welfare point of view.*

3.3 Comparison of Investment Levels

In this section we compare equilibrium investments in the scenarios we analyzed in the previous two sections and discuss the impact of uncertainty on the ranking we find.

Theorem 3 *(i) For any finite number of firms, n, it holds that*

- *Strategic firms invest less if they anticipate a more competitive result at the spot market, i. e. $X_n^C \geq X_n^{MC}$.*
- *Strategic firms invest too little from a social welfare point of view, i. e. $X_n^{SB} > X_n^C$.*
- *The first best solution yields the highest investment among all scenarios.*

Summarizing, it holds that $X_n^{FB} \geq X_n^{SB} > X_n^C \geq X_n^{MC}$.

(ii) As the number of firms approaches infinity, investment levels in all scenarios coincide, i. e. $X_\infty^{FB} = X_\infty^{SB} = X_\infty^C = X_\infty^{MC}$.

Proof See appendix D □

Let us briefly provide some intuition for our result, using some characteristics of the first order conditions as stated in theorems 1 and 2. Let us first draw the reader's attention to the particular structure of the first order conditions in theorems 1 and 2. They all equalize expected marginal profit or welfare [LHS] with marginal cost of capacity [RHS]. Note that, at the LHS, the stage one–objective (either profit or welfare) is reflected only in the integrand. That is, we integrate over *marginal profit* in cases where the firms maximize profits at stage one (C and MC) and over *marginal welfare* in cases where welfare is the stage one-objective (FB and SB). The stage two–objective enters exclusively into the lower limit of integration, which is the demand scenario from which on firms are constrained given the capacities chosen at stage one.

Now consider the optimal capacity choice of strategic firms. If the firms anticipate Cournot competition at stage two, marginal profit generated by additional capacity is positive in each scenario where the firm is constrained. If firms expect competitive behavior at the spot market, however, this is not the case. A firm might be forced to use additional capacity although the marginal profit from using it may be negative.[14] Consequently, additional capacity is less valuable to the firms in the latter case and investments are lower.

Comparison of the first order conditions in cases C (theorem 1) and SB (theorem 2) reveals why strategic firms always invest too little. Note that for any fixed capacity level, additional capacity is more valuable in case SB than in case C, since expected marginal welfare is always higher than expected marginal profit.[15] Consequently, from a social welfare perspective, strategic investments are too low.

Also the comparison of the First Best and the Second Best case is intuitive. As already mentioned, firms are constrained earlier the more competitive the spot market behavior is. Thus, in the First Best case, for any initial capacity level additional capacity is used more often and therefore generates a higher increase in social welfare.

Finally we derive exact conditions under which the weak inequalities from theorem 3 are strict, and hold with equality, respectively. They hold with equality whenever already the capacity choice determines production in any demand scenario θ where $f(\theta) > 0$, that is, if firms are always constrained at the production stage. Our theorem illustrates under what conditions on the nature of uncertainty the regime at stage two matters for the capacity choice (and when it is irrelevant).

Theorem 4 (Degenerate Cases) *Denote by $\underline{\theta}$ ($\overline{\theta}$) the lowest (highest) demand scenario where $f(\theta) > 0$ and suppose that $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Denote by $Q^C(\theta)$ the aggregate Cournot quantity in scenario θ in the absence of capacity constraints. It holds that*[16]

(i) $X^C \leq Q^C(\underline{\theta}) \quad \Leftrightarrow X^C = X^{MC}$,

(ii) $X^{FB} \leq Q^C(\underline{\theta}) \quad \Leftrightarrow X^{FB} = X^{SB}$.

Proof See appendix E □

The following table visualizes the result of theorem 4.

If condition (i) of theorem 4 holds, in the Cournot market game (see theorem 1) firms want to be constrained at the production stage in any state of nature $\theta \in [\underline{\theta}, \overline{\theta}]$. Since the incentive to be constrained is higher in case of optimal regulation at stage two, the solutions

[14] This is the case in all demand scenarios in $[\tilde{\theta}^{MC}, \tilde{\theta}^{C}]$.

[15] Formally, at a fixed capacity level, the critical value $\tilde{\theta}$ is the same in both cases, but the integrand is pointwisely bigger in case SB than in case C.

[16] The assumption $f(\theta) > 0$ is only needed for the "$\Leftarrow$"-direction. "$\Rightarrow$" always holds.

<table>
<tr><th>Genuine Uncertainty</th><th colspan="2">Degenerate Cases</th></tr>
<tr><td>$Q^C(\underline{\theta}) < X^C$</td><td>$X^C \leq Q^C(\underline{\theta}) < X^{FB}$</td><td>$X^{FB} \leq Q^C(\underline{\theta})$</td></tr>
<tr><td>$X^{MC} < X^C$</td><td colspan="2">$X^{MC} = X^C$</td></tr>
<tr><td colspan="2">$X^{SB} < X^{FB}$</td><td>$X^{SB} = X^{FB}$</td></tr>
</table>

Table 2: Degenerate Cases and Equivalence of Scenarios.

of C and MC collapse in this case. Moreover, comparison with a result by Reynolds and Wilson (2000) shows that under condition (i) also a game where firms invest prior to Bertrand competition at stage two yields the same capacity as C and MC.[17] Obviously, condition (i) describes a degenerate environment where uncertainty does not matter much. Under genuine uncertainty, where firms are unconstrained in at least some states of nature, our analysis demonstrates that in fact market organization at stage two matters a lot.[18]

If condition (ii) holds, at the welfare maximizing (First Best) capacity level even strategic firms are constrained in any demand scenario $\theta \in [\underline{\theta}, \overline{\theta}]$ at stage two. Notice that condition (ii) is stronger than condition (i) [since $X^{FB} > X^C$, as we have shown in theorem 3]. Consequently, (ii) can only hold in a degenerate environment where uncertainty is not an important issue.

The reason why the level of uncertainty is not the only decisive factor for a equivalence of X^{FB} and X^{SB} can best be illustrated in case of certain demand. At the production stage, strategic firms play either their Cournot quantity given marginal *production* cost, or their capacity, whichever is lower. This implies that even under certainty the First Best and the Second Best outcome coincide only in those cases where the First Best capacity level is *below* the Cournot quantities at stage two. Thus, condition (ii) requires that marginal capacity cost is sufficiently high compared to marginal production cost and that uncertainty does not matter much. As we have shown in our analysis, however, under genuine uncertainty

[17]Reynolds and Wilson show that under condition (i) capacity choice prior to Bertrand competition yields the same outcome as capacity choice in a game where firms cannot adjust their production after uncertainty unraveled. It is easy to show that under condition (i) the latter game yields the same outcome as our Cournot market game (which clearly is not the case if condition (i) does not hold).

[18]For the Bertrand market game Reynolds and Wilson (2000) show that under genuine uncertainty equilibria with equal capacities of the firms do not exist.

the First Best solution always implies higher investment than the second best solution, independent of marginal capacity and production cost.

Whereas capacities in the four scenarios we analyze can be ranked unambiguously, this is not always true when it comes to social welfare. A welfare comparison is simple and straightforward for cases C, SB, and FB (where welfare is increasing in this order). It is not obvious, however, whether welfare is higher in case C or MC. In scenario C firms exercise market power at the spot market, whereas in case MC spot prices are regulated to the competitive level. Thus, in absence of capacity constraints welfare would be higher in MC. However, at stage one strategic firms choose lower capacities in case MC such that prices are higher in case MC than in C whenever firms are capacity constrained in both cases. Consequently, a welfare comparison between the two cases is not straightforward and necessarily depends on details of the model's specification. A simplified model with linear demand demonstrates that both, an increase and a decrease in welfare is possible and suggests that regulation of the production stage is particularly undesirable from a welfare point of view if the number of firms is low. We come back to this issue in section 4, where we fit our model to the data of the German electricity market.

4 An Empirical Analysis of Capacity Choice in Electricity Markets — The Example of Germany

In this section we demonstrate how our theoretical insights can be used to assess (long run) capacity and welfare effects of electricity market liberalization. We also quantify the capacity and welfare effects of several recent policy proposals for different degrees of market concentration.[19] The approach can be applied to any electricity market by fitting the theoretical model to the corresponding data and comparing predicted strategic capacity choices to the actually installed level.[20] Here, for the reason of data availability, we use data of the German electricity market.

[19] All welfare effects we demonstrate can also be shown in a simplified model with linear demand and uniform distribution of uncertainty. In particular, in Grimm and Zoettl (2007) we show that the more concentrated the market is, the less competitive the stage–two market outcome should be from a welfare point of view.

[20] We are not aware of any empirical studies of investment in electricity markets. The main reason presumably is that the post liberalization period is not yet long enough to generate data on investment cycles. This is also a strong argument for fitting a theoretical model to the primitives of a market to get an impression of possible long run effects that cannot appear in the data yet.

Note that — although they are quite stylized — our scenarios capture nicely some recent policy proposals. A spot market intervention as described in case MC is closely related to the common proposal to monitor tightly the firms' spot market behavior.[21] The difference of capacity levels in scenarios C and SB is a proxy for the desirability of capacity markets or other mechanisms that increase investment incentives. Thus, our analysis yields insights to assess some policy tools that have been at the focus of the current debate on the need of reorganization of the German electricity market. Apart from capacity choices, we also focus on the price distribution in the different scenarios and on welfare implications of regulatory interventions.

Our aim is to fit the theoretical model as closely as possible to the data of the German Electricity market for the year 2006 and to compute resulting investment in gas turbine generation capacity for the scenarios MC, C, SB, and FB. Note that this approach yields total investment under the assumption that each firm's marginal generating unit is always a gas turbine. Since investment in the last unit of capacity (which, of course, determines total capacity) is always a marginal decision, we do not need to specify the inframarginal technology mix for the empirical analysis. Note however, that we need to assume that firms are symmetric in size (but not necessarily with respect to their inframarginal technology mix). Since mark-ups in the Cournot model generally increase if firms become asymmetric, our results yield a lower bound for the extent of market power for a given number of firms.

In order to use our theoretical model for the analysis we chose to make the following specifications. We assume linear fluctuating demand $P(Q) = \theta - bQ$ and fluctuating but constant marginal cost $c(\theta)$. Note that for linear demand our model can allow simultaneously for both, demand and cost uncertainty. If we sort all realizations of demand and cost according to the differences $\theta - c(\theta)$, the resulting framework satisfies assumptions 1 to 3. Furthermore, for the sake of our applied example, we interpret the distribution over the demand scenarios as relative frequencies which have been accurately predicted by all firms.[22]

For a given demand and cost distribution and for given marginal investment cost, pre-

[21] See, e.g. Monopolkommission (2007), p.4, paragraph 9.*. If the regulator has perfect information, the result of such an intervention would be marginal cost pricing at stage two.

[22] That is, in our empirical analysis we have no uncertainty but just demand fluctuation over time. In practice, there should be two competing effects if uncertainty would be added to the analysis. Since investment in gas turbines is rather risky and firms are typically risk averse, the benchmark determined should yield too much investment. On the other hand, however, our model implies that a risk neutral firm should invest more if risk is increased.

dicted capacities can be calculated by solving the corresponding first order conditions as stated in theorems 1 and 2. The resulting capacity choices allow us to derive the price distribution for those hours where capacity is binding, and to compare it to the observed price distribution. Moreover, we can capture the welfare effect of regulatory interventions. To this aim we calculate the welfare difference to the Cournot case for scenarios MC, SB, and FB and add up welfare differences generated in each hour of the year.

In order to assess the robustness of our results we do not perform the analysis for single parameter values, but rather for plausible ranges of parameter distributions. This concerns the following parameters of the model: The demand elasticity (determined by the slope of the demand function, b), marginal cost of production, c, and marginal investment cost, k. From the possible ranges of those parameters, our algorithm selects one random combination in each iteration. The resulting distributions of capacities and welfare differences give an impression of the sensitivity of our results to changes in the parameters. In the following we provide some details on the relevant ranges of our cost and demand parameters.

Market demand: To construct fluctuating market demand, we depart from hourly market prices (from the European Energy Exchange (EEX)[23]) and hourly quantities consumed (from the Union for the Co-ordination of Transmission of Electricity (UCTE)[24]) for the year 2006. We chose the value of b in line with other studies on energy markets. Most studies that estimate demand for electricity[25] find short run elasticities between 0.1 and 0.5 and long run elasticities between 0.3 and 0.7.[26] The relevant range of prices is around $P = 100$ €/MWh and corresponding consumption is approximately $Q = 50$ GW. In our simulations we thus use a uniform distribution of b on the interval $[0.004, 0.007]$, which corresponds to elasticities between 0.5 and 0.29.

Production cost: The major components of variable production cost are gas prices[27] and prices for CO_2 emission allowances.[28] The average TTF gas price in 2006 was 20

[23]See www.EEX.com

[24]See www.UCTE.org

[25]See, for example, Lijsen (2006) for an overview of recent contributions on that issue.

[26]E.g. Beenstock et al. (1999), Bjorner and Jensen (2002), Filippini Pachuari (2002), Booinekamp (2007), and many others.

[27]Daily values from the Dutch Hub TTF, corrected for transportation cost.

[28]Daily data taken from the EEX. The emission-coefficient for natural gas is set by the German ministry of environment at 56t CO_2/TJ which corresponds to 0.2016t CO_2/MWh. Compare Umweltbundesamt (2004).

€/MWh and CO_2 permissions traded on average for 9.30 €/MWh.[29] The efficiency of gas turbines currently ranges at around $37,5\%$.[30] The resulting daily production cost for the year 2006 was on average 66.30 €/MWh. Daily values, as used in our empirical analysis, are illustrated in figure 1. In our simulations we use the observed distribution but multiply each realization by the factor f which is uniformly distributed in [0.9, 1.1].

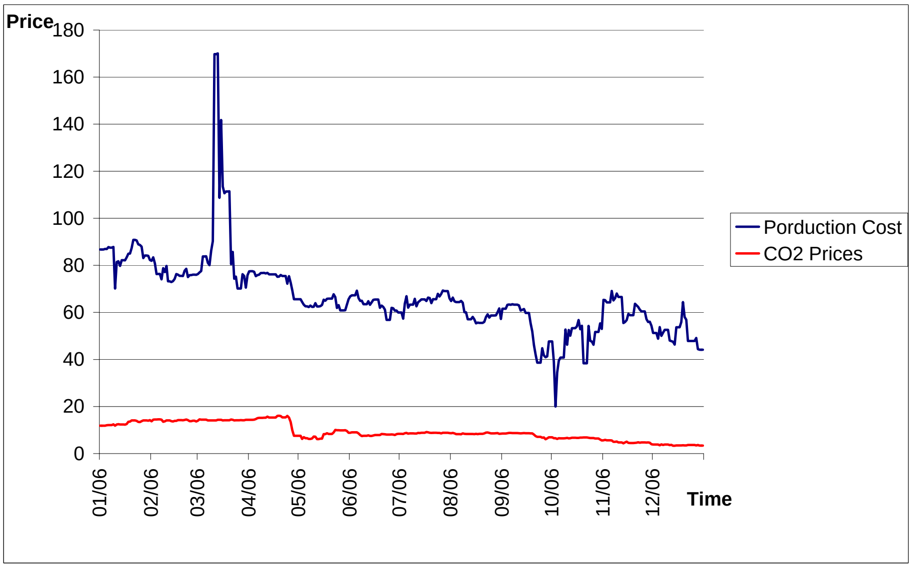

Figure 1: Production Cost in the Year 2006.

Investment Cost: Since we analyze investment incentives based solely on one year, we break down investment cost to annuities.[31] In order to take construction time of gas turbine plants into account we consider investment cost on the basis of data from the year 2000. We assume perfect foresight, i.e. all cost components have been predicted accurately by the firms at the time of their investment decision. We base investment cost on the following

[29] Recall that we do not use the averages but the daily values in our simulation.

[30] See 2006 GTW Handbook or EWI and Prognos (2005).

[31] The results will thus only yield a benchmark for current profitability of investment. Provided, however, that yearly demand is increasing over time (and that strategic timing of investment is not an issue) our procedure should yield accurate predictions, even though once installed capacities cannot be removed the subsequent year.

two studies: First, a study on the German energy market commissioned by the German Parliament (2002), with scenarios for investment decisions summarized in Weber and Swider (2004) [in the following GP/WS]. Second, Energiereport III, a study conducted by the Institute of Energy Economics (EWI) in Cologne and Prognos (2000) for the the German Ministry of Economics [in the following EWI/P].

The relevant annuity is determined as follows: Total investment cost ranges between 279 €/KW (GP/WS) and 300 €/KW (EWI/P). Annual fixed cost of running a gas turbine is already included in GP/WS, and is given by 8 €/KWa in EWI/P. This value is corrected by the average availability of gas turbines, which, in Germany, is given by 94%.[32] Based on a financial horizon of 20 years and an interest rate of 10 % this yields annuities of 34863 €/MWa (GP/WS) and 45998 €/MWa (EWI/P). Finally, the free allotment of CO_2 allowances granted to new power plants results in a de facto reduction of the annuity by the net value of the allocated allowances. Calculating their value on the basis of the average market price in 2006 yields 6305.3 €/MWa. The range of relevant annuities which we use in our simulation is consequently given by [28558, 39692] €/MWa.

Figure 2 shows — for different numbers of firms — total investment in all four scenarios we discuss. In the figure, the big symbols represent the average value while the two smaller symbols of the same type determine the 90 % confidence interval of our simulation. Obviously, predicted capacities are not very sensitive to changes in the parameters. The first best investment does not change in the number of firms since we assume that each firm's marginal generating unit is a gas turbine, independent of the number of firms and the level of demand. Strategic capacity choice (scenario C) is at only 50 % of the optimal level for the monopoly case, while it is at 80 % of the optimal level for four firms. The graph illustrates that the presence of market power not only affects spot prices, but also has a strong effect on capacity choices. Total capacity installed in Germany in 2006 was approximately 68 GW in a market with four large firms.[33] The relatively high level of actual capacity as compared to our results reflects the fact in the pre-liberalization period (i.e. before 1998) generators where subject to a rate of return regulation that imposed excessive investment incentives.

From the predicted capacity levels we now compute the price distribution for those hours where capacity is predicted to be binding in the Cournot game. Since we want to

[32]Compare VGB Powertech (2006).

[33]The German market consists essentially of four large players. Two of them (RWE and E.on) have a market share of 26 % each, while the two smaller ones (ENBW and Vattenfall) together cover 30 % of the market each. Compare, e.g., Monopolkommission (2007).

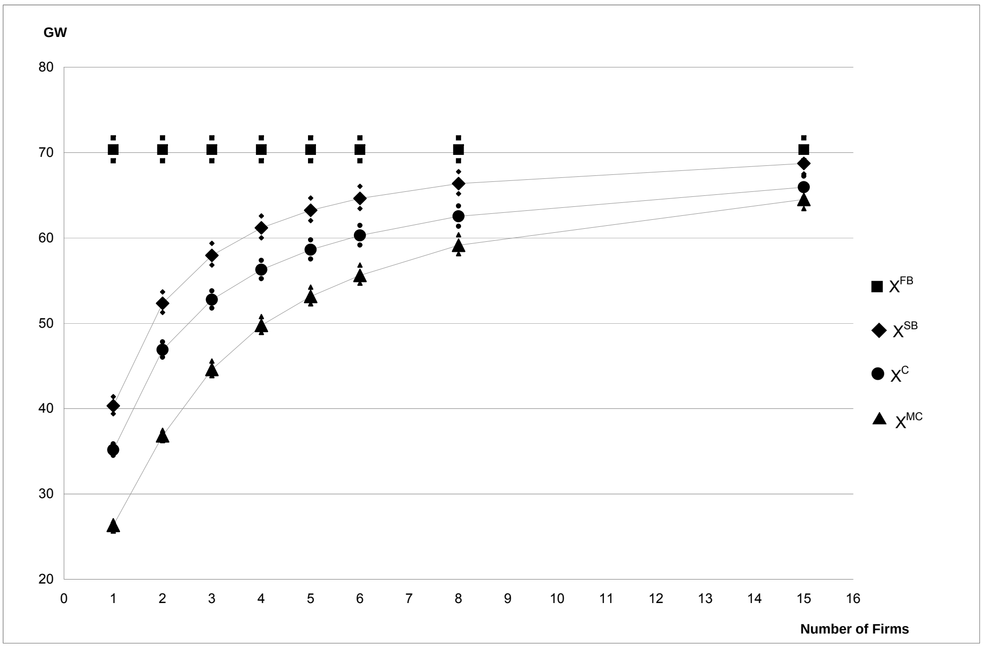

Figure 2: Investment Levels in all Four Cases.

compare predicted prices to the observed price distribution, we choose (in accordance with the German market structure) a scenario of four firms. We, moreover, choose the mean values of the parameter intervals which we used in our simulations, i.e. $b = 0.0055$, and $k = 35430/MWa$.[34] For our data set strategic firms are capacity constrained in approximately 1107 hours (12.6 % of the year).[35] Figure 3 provides the observed price distribution (grey line), as well as the predicted price distributions during the hours with a binding capacity constraint, separately for scenarios FB, SB, C, and MC (black lines). In order to make the differences more visible, in the figure we focus on prices in the interval $[0, 500]$ and provide information on the highest price realizations in the legend. Obviously, for the parameter configuration we chose, observed prices are above predicted prices in the first best scenario

[34] We could also determine the price distribution for ranges of parameters. Since capacities have turned out not to be very sensitive to changes in the parameters, however, we chose to use mean values to make our illustration more readable.

[35] Our predicted values match the empirical observations. Due to Umweltbundesamt (2004), gas turbines run approximately 10 % of the time.

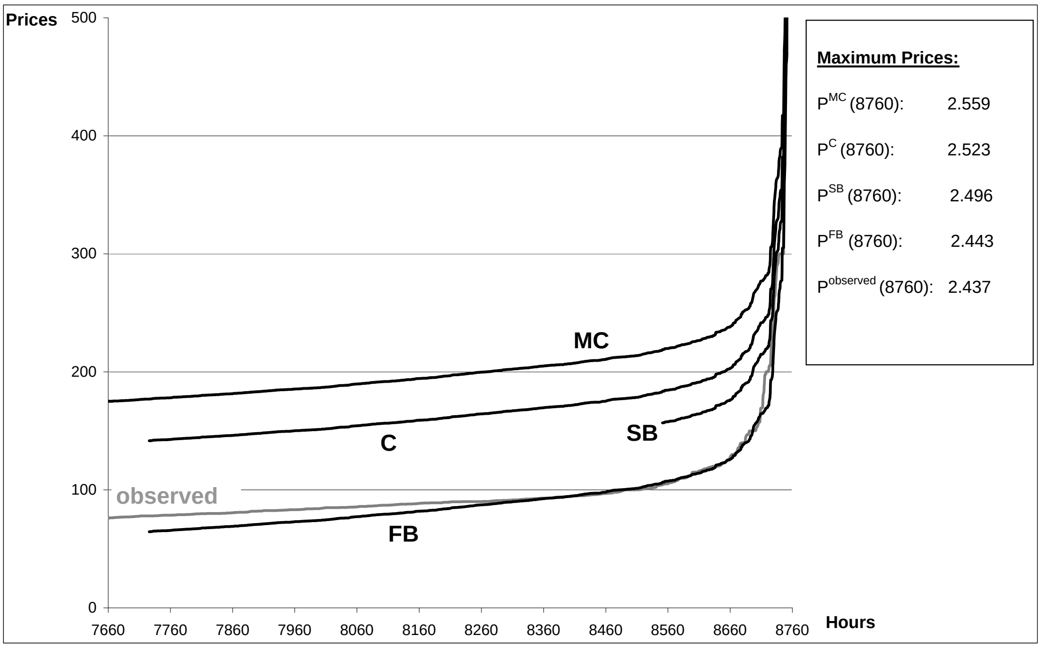

Figure 3: Price Distribution in the Hours where Capacity is Binding, Cases C, MC, SB, FB, and Observed Prices.

but well below predicted prices in the Cournot market game. All depicted prices reflect the willingness to pay for an additional unit of capacity that cannot be produced in the short run. Notice that the relatively low level of observed prices (as compared to the Cournot scenario) may well be due to the fact that currently firms have more capacity installed than they would have chosen in a liberalized regime.[36] Our theoretical analysis implies that the current prices do not yield sufficient investment incentives to sustain the current capacity level. Strategic investment would strongly affect the price distribution, as comparison of the curves for the cases FB and C illustrates. Obviously, there is a strong potential for market power not only in the short run, but also at the investment stage.

Finally, figure 4 illustrates the welfare effect that results from regulation of spot market prices down to the competitive level. All welfare differences are calculated in relation to the

[36]In the pre-liberalization period, generators where subject to a rate of return regulation that imposed excessive investment incentives.

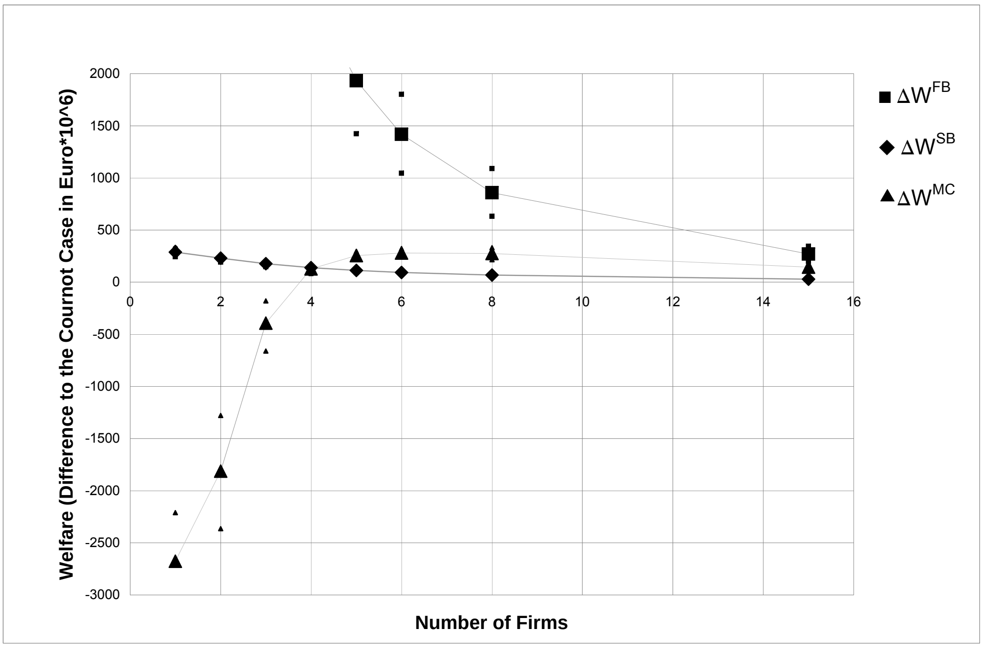

Figure 4: Welfare Differences to the Cournot Case (C) for Cases MC, SB, and FB.

Cournot market game. Again, we ran simulations using the relevant parameter ranges. Big symbols represent average welfare differences while small symbols are the 90 % confidence intervals. As we have already seen from the theoretical analysis and from figure 2, imposing marginal cost prices at the spot market considerably decreases equilibrium investment. The figure shows that if the number of firms in the market is low, enforcement of marginal cost pricing at the spot market moreover decreases total welfare. Only if the number of firms is four or higher, total welfare is increasing. Thus, our analysis demonstrates that intervention only at the spot market does not necessarily have the desired effect if firms choose their capacities strategically.

The figure moreover illustrates the welfare effect of intervention only at the investment stage (scenario SB) and of implementation of the First Best solution. As it becomes clear from the graph, performance of the Cournot market game is getting very close to the first best solution as the number of competitors becomes large. We also observe that, while the effect of increasing capacities given that firms have market power at the spot market is moderate for all market structures, intervention at the spot market may have relatively large negative effects on welfare if the number of firms is low.

5 Conclusion

In this paper we have provided a general model of strategic investment decisions under uncertainty prior to imperfectly competitive markets. We have shown existence and uniqueness of equilibrium and provided an intuitive characterization of equilibrium investment. We found that under imperfect competition increasing capacity is desirable from a social welfare point of view. We also demonstrated that intervention only at the spot market leads to strategic uncertainty at the investment stage and, moreover, decreases total investment. Thus, in markets with considerable demand fluctuations, (partial) intervention only at the spot market stage has to be carefully reconsidered.

We have also fitted our theoretical model to the data of the German electricity market. We derive predicted capacity levels for various degrees of market concentration, and illustrated welfare effects of regulatory interventions. In a market of four firms (which corresponds to the current situation in Germany) predicted strategic capacity choices are at 80 % of the First Best level, while installed capacity is even at approximately 96 % of our First Best prediction. This is presumably due to high investment incentives in the pre–liberalization period. In accordance with the relatively high current capacity level, the observed distribution of prices in 2006 is close to the predicted First Best price distribution for those scenarios where our model predicts that capacity is binding. An immediate implication of our theoretical analysis is, however, that the observed prices are not high enough to sustain the current level of capacity if investment is strategic. Moreover, for a market structure of four firms we find a slightly positive welfare effect of market monitoring at the production stage. For highly concentrated markets (i.e. monopoly or duopoly), strategic capacity choices are far below the First Best level. We find that in concentrated markets, market monitoring at the spot market would decrease the investment incentives drastically and would therefore have a large and negative welfare effect.

While the model provides a solid intuition for how investment incentives and welfare are affected by regulatory intervention, specific market designs under consideration still have to be analyzed carefully in order to obtain reliable policy conclusions. To this aim, our model provides a tractable framework for the analysis of different scenarios at the market stage. The framework captures the stylized fact that at the time when they make their investment decisions firms face considerable uncertainty both about future demand and production cost, and probably also with respect to future regulatory regimes. Let us outline several directions of research that can directly benefit from the analysis done in this paper.

The most obvious application of the model is to modify the game played at the second stage in order to analyze how different market designs or regulatory interventions affect investment incentives and welfare. In this line, Grimm and Zoettl (2006) analyze how investment incentives are affected by the introduction of forward markets prior to spot trading. Another closely related article, Grimm and Zoettl (2007), uses the present framework to analyze the effect of price caps on production and welfare under demand uncertainty. Whereas the results of Grimm and Zoettl (2006) confirm the intuition that making the spot market outcome more competitive (through the introduction of forward markets) decreases investments, Grimm and Zoettl (2007) find that price caps at stage two may actually increase investment incentives. The reason is that price caps eliminate an important feature of the present model, i. e. prices cannot rise unboundedly in case of insufficient capacity, which makes strategic withholding of capacity less profitable.

A second line of research for which the current model serves as a starting point is the analysis of choice between different production technologies. On the one hand, such a model would allow to analyze the effect of policy tools like emission allowances in electricity markets on the technology mix chosen by strategic firms. On the other hand it could serve as the theoretical benchmark that allows to estimate the effect of market power at the investment stage also for inframarginal technologies.

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A Analysis of the Production Stage

The appendix contains all proofs of the paper. In the first part, we analyze the second stage of the game, which we need in order to proof theorems 1 (appendix B) and 2 (appendix C).

In the first step we characterize capacity constrained production choices at stage two for each θ given investment choices x. Note that we have to consider also asymmetric investments. In order to simplify the exposition we will order the firms according to their investment levels, i. e. $x_1 \leq x_2 \leq \cdots \leq x_n$, throughout the paper. At stage two either firms engage in Cournot competition or a social planer implements the optimal production schedule given investment choices. In this appendix we analyze both scenarios.

A.1 Cournot Competition at the Production Stage

An equilibrium of the capacity constrained Cournot game at stage two in scenario θ given x, $q^C(x,\theta)$, satisfies simultaneously for all firms

$$q_i^C(x,\theta) \in \arg\max_{\mathrm{q}} \left\{ P(\mathrm{q} + q_{-i}^C, \theta))\mathrm{q} - C(\mathrm{q},\theta) \right\} \qquad \text{s.t.} \quad 0 \le \mathrm{q} \le x_i. \tag{3}$$

Note that at very low values of θ all firms are necessarily unconstrained. By assumption 1 the unconstrained Cournot equilibrium [which we denote by $\tilde{q}^{C0}(\theta)$] is unique and symmetric for each $\theta \in [-\infty,\infty]$.[37] From (3) it follows that $\tilde{q}_i^{C0}(\theta)$ is implicitly determined by the first order condition

$$P(n\tilde{q}_i^{C0},\theta) + P_q(n\tilde{q}_i^{C0},\theta)\tilde{q}_i^{C0} = C_q(\tilde{q}_i^{C0},\theta).$$

Now as θ increases, at some critical value that we denote by $\theta^{C1}(x)$, firm 1 (the one with the lowest capacity) becomes constrained. The critical demand scenario is implicitly determined by $x_1 = q_1^{C0}(\theta^{C1})$. If it holds that $x_1 < x_2$, then at $\theta^{C1}(x)$ only firm one becomes constrained. Then, in equilibrium, firm 1 produces at its capacity bound whereas the remaining firms produce their equilibrium output of the Cournot game among $n-1$ firms given the residual demand $P(Q - x_1, \theta)$ [denoted by $\tilde{q}_i^{C1}(x,\theta)$], which solves the first order condition

$$P(x_1 + (n-1)\tilde{q}_i^{C1},\theta) + P_q(x_1 + (n-1)\tilde{q}_i^{C1},\theta)\tilde{q}_i^{C1} = C_q(\tilde{q}_i^{C1},\theta).$$

The capacity constrained Cournot equilibrium in the case where one firm is constrained is a vector $q^{C1}(x,\theta)$, where $q_i^{C1}(x,\theta) = \min\{x_i, \tilde{q}^{C1}(x,\theta)\}$.

As θ increases further, we pass through $n+1$ cases, from case $C0$ (no firm is constrained) to case Cn (all n firms are constrained). Note that two critical values $\theta^{Cm}(x)$ and $\theta^{Cm+1}(x)$ coincide whenever $x_m = x_{m+1}$, and that it holds that $\theta^{Cm}(x) < \theta^{Cm+1}(x)$ (by assumption 2) whenever $x_m < x_{m+1}$.

Now we are prepared to characterize the capacity constrained Cournot equilibrium in case Cm where m firms are constrained. In this case, the m firms with the lowest capacities produce at their capacity bound, whereas the $n-m$ unconstrained firms produce

$$\begin{aligned}\tilde{q}_i^{Cm}(x,\theta) \quad = \quad & \Bigg\{ q_i \in \mathbb{R} : P\left(\sum_{i=1}^{m} x_i + (n-m)\,\tilde{q}_i^{Cm}, \theta\right) \\ & \quad + P_q\left(\sum_{i=1}^{m} x_i + (n-m)\,\tilde{q}_i^{Cm}, \theta\right)\tilde{q}_i^{Cm} = C_q\left(\tilde{q}_i^{Cm},\theta\right)\Bigg\},\end{aligned} \tag{4}$$

[37] See, for example Selten (1970), or Vives (2001), pp. 97/98.

The equilibrium quantities of the capacity constrained Cournot game in case Cm are given by

$$q_i^{Cm}(x,\theta) = \min\{x_i, \tilde{q}_i^{Cm}(x,\theta)\}, \tag{5}$$

and aggregate production in case Cm is

$$Q^{Cm}(x,\theta) = \sum_{i=1}^{n} q_i^{Cm}(x,\theta). \tag{6}$$

This allows us finally to pin down the profit of firm i in scenario Cm,

$$\pi_i^{Cm}(x,\theta) = \begin{cases} P\left(Q^{Cm},\theta\right) x_i - C\left(x_i,\theta\right) & \text{if } i \leq m, \\ \\ P\left(Q^{Cm},\theta\right) \tilde{q}_i^{Cm}(x,\theta) - C\left(\tilde{q}_i^{Cm}(x,\theta),\theta\right) & \text{if } i > m. \end{cases} \tag{7}$$

Note that it holds that $\frac{d\pi_i^{Cm}}{dx_i} > 0$ only if $i \leq m$, and $\frac{d\pi_i^{Cm}}{dx_i} = 0$ otherwise, since a firm's capacity expansion only affects production at stage two in case the firm was constrained. Obviously, in this case the derivative must be positive.

We can finally pin down maximal social welfare generated in demand scenario $\theta \in [\theta^{Cm}, \theta^{Cm+1}]$ (where, given x, the m lowest capacity firms are constrained) as

$$W^{Cm}(x,\theta) = \int_0^{Q^{Cm}(x,\theta)} P(Q,\theta)\, dQ - \sum_{i=1}^{n} C\left(q_i^{Cm}(x,\theta),\theta\right). \tag{8}$$

(we need this in order to prove Part (SB) of theorem 2). Note that W^{FBm} only depends on x_i if firm i is constrained in scenario m, that is if $i \leq m$.

Property 1 (Monotonicity of θ^{Cm}) *$\frac{d\theta^{Cm}(x)}{dx_i}$ is strictly positive if $i \leq m$ (i.e. if firm i produces at its capacity bound), and zero otherwise.*

Proof $\theta^{Cm}(x)$ is the demand realization from which on firm m cannot play its unconstrained output any more. At $\theta^{Cm}(x)$ it holds that $q_i^C(\theta^{Cm}(x)) = \tilde{q}_i^{Cm}(\theta^{Cm}(x)) = x_m$ for all $i \geq m$ and $q_i^C(\theta^{Cm}(x)) = x_i < x_m$ for all $i < m$. Thus, $\theta^{Cm}(x)$ is implicitly defined by the conditions

$$\begin{aligned} & P\left(\sum_{i=1}^{m} x_i + (n-m)x_m, \theta^{Cm}(x)\right) \\ & \quad + P_q\left(\sum_{i=1}^{m} x_i + (n-m)x_m, \theta^{Cm}(x)\right) x_m - C_q\left(x_m, \theta^{Cm}(x)\right) = 0. \end{aligned}$$

Differentiation with respect to x_i, $i < m$, yields

$$P_q(\cdot) + P_\theta(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} + P_{qq}(\cdot)\, x_m + P_{q\theta}(\cdot)\, x_m \frac{d\theta^{Cm}(x)}{dx_i} - C_{q\theta}(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} = 0,$$

and solving for $\frac{d\theta^{Cm}(x)}{dx_i}$ we obtain

$$\frac{d\theta^{Cm}(x)}{dx_i} = -\frac{P_q(\cdot) + P_{qq}(\cdot)\, x_m}{P_\theta(\cdot) + P_{q\theta}(\cdot)\, x_m - C_{q\theta}(\cdot)} > 0$$

due to assumption 1, part (i) and assumption 2, part (ii) [note that the expression in the denominator is the cross derivative which was assumed to be positive in part (ii) of assumption 2].

Differentiation with respect to x_i, $i = m$, yields

$$\begin{aligned}&(n-m+2)P_q(\cdot) + P_\theta(\cdot)\frac{d\theta^{Cm}(x)}{dx_i}\\ &\quad +(n-m+1)P_{qq}(\cdot)\, x_m + P_{x\theta}(\cdot)\, x_m\frac{d\theta^{Cm}(x)}{dx_i} - C_{xx}(\cdot) - C_{q\theta}(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} = 0,\end{aligned}$$

and solving for $\frac{d\theta^{Cm}(x)}{dx_i}$ we obtain

$$\frac{d\theta^{Cm}(x)}{dx_i} = -\frac{(n-m+2)P_q(\cdot) + (n-m+1)P_{qq}(\cdot)\, x_m - C_{xx}(\cdot)}{P_\theta(\cdot) + P_{q\theta}(\cdot)\, x_m - C_{q\theta}(\cdot)} > 0,$$

also due to assumption 1, parts (i) and assumption 2, part (ii). Finally, differentiation with respect to x_i, $i > m$, yields

$$P_\theta(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} + P_{x\theta}(\cdot)\, x_m\frac{d\theta^{Cm}(x)}{dx_i} - C_{q\theta}(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} = 0,$$

which implies that $\frac{d\theta^{Cm}(x)}{dx_i} = 0$ for $i > m$. □

A.2 Welfare maximization at the Production stage

In the following we specify, for a given vector of capacities x, the welfare optimal production schedule for any possible demand scenario (that is, for any possible value of θ).[38]

Note that necessarily all firms are unconstrained for very low values of θ. It is straightforward to show that in the welfare optimum, all unconstrained firms produce the same (due to convex cost). Thus, the socially optimal total quantity of each firm if all firms are unconstrained is given by $q_i^{FB0}(\theta) = \{q_i \in \mathbb{R} : P(nq_i, \theta) = C_q(q_i, \theta)\}$.

Now, as θ increases, at some critical value, that we denote by $\theta^{FB1}(x)$, firm 1 (the lowest capacity firm) becomes constrained. The critical demand scenario $\theta^{FB1}(x)$ is implicitly defined by $x_1 = q_1^{FB0}(\theta^{FB1})$. If it holds that $x_1 < x_2$, then at $\theta^{FB1}(x)$ only firm 1 becomes constrained and the socially optimal production plan implies that firm 1 produces at its capacity bound whereas the remaining firms produce the unconstrained optimal quantity given the

[38] With convex cost a characterization of the welfare optimum could probably be given with less mathematical burden. However, we will need the characterization developed here also in section **??**.

residual demand $P(Q - x_1, \theta)$, i. e. $\tilde{q}_i^{FB1}(x,\theta) = \{q_i \in \mathbb{R} : P((n-1)q_i + x_1, \theta) = C_q(q_i, \theta)\}$. The optimal production plan in scenario $FB1$ is a vector $q^{FB1}(x,\theta)$, where each element is given by $q_i^{FB1}(x,\theta) = \min\{x_i, \tilde{q}_i^{FB1}(x,\theta)\}$.

As θ increases further and more firms become constrained, we pass through $n+1$ cases, from case $FB0$ (no firm is constrained) to case FBn (all n firms are constrained). Note that two critical values $\theta^{FBm}(x)$ and $\theta^{FBm+1}(x)$ coincide whenever $x_m = x_{m+1}$, and that it holds that $\theta^{FBm}(x) < \theta^{FBm+1}(x)$ (by assumption 2) whenever $x_m < x_{m+1}$.

Now we are prepared to characterize the socially optimal production plan and social welfare generated in case FBm, where m firms are constrained. In this case, the m firms with the lowest capacities produce at their capacity bound, whereas the $n-m$ unconstrained firms produce the unconstrained optimal quantity given the residual demand $P(Q - \sum_{i=1}^m x_i, \theta)$, i. e.

$$\tilde{q}_i^{FBm}(x,\theta) = \left\{ q_i \in \mathbb{R} : P\left(\sum_{j=1}^m x_j + (n-m)q_i, \theta \right) = C_q(q_i,\theta) \right\}. \tag{9}$$

We denote the optimal production plan in case FBm by $q^{FBm}(x,\theta)$ where each element is given by

$$q_i^{FBm}(x,\theta) = \min\{x_i, \tilde{q}_i^{FBm}(x,\theta)\} \qquad i = 1,\ldots,n. \tag{10}$$

Consequently, the optimal total quantity produced in case FBm is

$$Q^{FBm}(x,\theta) = \sum_{i=1}^n q_i^{FBm}(x,\theta). \tag{11}$$

This allows to pin down firm i's profit in scenario FBm,

$$\pi_i^{FBm}(x,\theta) = \begin{cases} P\left(Q^{FBm}(x,\theta),\theta\right) x_i - C(x_i,\theta) & \text{if} \quad i \le m, \\ \\ P\left(Q^{FBm}(x,\theta),\theta\right) \tilde{q}_i^{FBm}(x,\theta) - C\left(\tilde{q}_i^{FBm}(\cdot),\theta\right) & \text{if} \quad i > m. \end{cases} \tag{12}$$

We can finally pin down maximal social welfare generated in demand scenario $\theta \in [\theta^{FBm}, \theta^{FBm+1}]$ (where, given x, the m lowest capacity firms are constrained) as

$$W^{FBm}(x,\theta) = \int_0^{Q^{FBm}(x,\theta)} P(Q,\theta)\, dQ - \sum_{i=1}^n C\left(q_i^{FBm}(x,\theta),\theta\right). \tag{13}$$

(we need this in the proof of theorem 2). Note that W^{FBm} only depends on x_i if firm i is constrained in scenario m, that is if $i \le m$.

B Proof of Theorem 1

B.1 Proof of Theorem 1, Case (C)

Now we are prepared to analyze capacity choices at the investment stage. The results obtained for the production stage enable us to derive a firm i's profit from investing x_i, given that the other firms invest x_{-i} and quantity choices at stage two are given by $q^{Cm}(x,\theta)$ for $\theta \in [\theta^{Cm}(x), \theta^{Cm+1}(x)]$. Recall that when choosing capacities the firms face demand uncertainty. Thus, a firm's profit from given levels of investments, x, is the integral over equilibrium profits at each θ given x on the domain $[-\infty, \infty]$, taking into account the probability distribution over the demand scenarios. For each θ, firms anticipate equilibrium play at the production stage, which gives rise to one of the $n+1$ types of equilibria, EQ^{C0}, $\dots$, EQ^{Cm}, $\dots$, EQ^{Cn}. Note that any $x > 0$ gives rise to the unconstrained equilibrium if θ is sufficiently low. As θ increases, more and more firms become constrained. Thus, a tuple of investment levels that initially gave rise to an EQ^{C0}, then leads to an equilibrium where first one (then two, three, $\dots$, and finally n) firms are constrained. In order to simplify the exposition we define $\theta^{C0} \equiv -\infty$ and $\theta^{Cn+1} \equiv \infty$. Then, the profit of firm i is given by[39]

$$\pi_i(x, q^C) = \sum_{m=0}^{m=n} \int_{\theta^{Cm}}^{\theta^{Cm+1}} \pi_i^{Cm}(x,\theta) dF(\theta) - K(x_i). \tag{14}$$

Note that at each critical value θ^{Cm}, $m = 1, \dots, n$ it holds that $\pi^{Cm-1}(x, \theta^{Cm}) = \pi^{Cm}(x, \theta^{Cm})$. Thus, $\pi_i(x, q^C)$ is continuous. Differentiating $\pi_i(x, q^C)$ yields[40]

$$\frac{d\pi_i\left(x, q^C\right)}{dx_i} = \sum_{m=i}^{n} \int_{\theta^{Cm}(x)}^{\theta^{Cm+1}(x)} \frac{d\pi_i^{Cm}(x,\theta)}{dx_i} dF(\theta) - K_x(x_i) \tag{15}$$

We prove part (i) of the lemma in two steps. In part I we show existence and in part II uniqueness of the equilibrium.

Part I: Existence of Equilibrium In the following we show that a symmetric equilibrium of the two stage Cournot market game exists, and that equilibrium choices $x_i^C = \frac{1}{n} X^C$, $i = 1, \dots, n$, are implicitly defined by equation (2). For this purpose it is sufficient to show

[39] Note that it is never optimal for a firm to be unconstrained at ∞ and thus, we always obtain $\theta^{Cn} \leq \infty$.

[40] Note that continuity of π_i implies that due to Leibnitz' rule the derivatives of the integration limits cancel out. Moreover, π_i^{Cm} only changes in x_i if firm i is constrained in scenario FBm, i. e. $i \leq m$. Thus, the sum does not include the cases where firm i is unconstrained, i. e. $m < i$.

quasiconcavity of firm i's profit given the other firms invest x_{-i}^C, $\pi_i(x_i, x_{-i}^C)$, which we do in the following.

Note that $\pi_i(x_i, x_{-i}^C)$ is defined piecewisely. For $x_i < x_i^C$, we have to examine to profit of firm 1 (by convention the lowest capacity firm) given that $x_2 = x_3 = \cdots = x_n$. Since this implies that $\theta^{C2} = \cdots = \theta^{Cn}$ and thus it follows from (14) that

$$\begin{aligned}\pi_1(x_1, x_{-1}^C) \quad &= \quad \int_{-\infty}^{\theta^{C1}(x)} \pi_1^{C0}(x,\theta)dF(\theta) + \int_{\theta^{C1}(x)}^{\theta^{Cn}(x)} \pi_1^{C1}(x,\theta)dF(\theta) \\ &\quad + \int_{\theta^{Cn}(x)}^{\infty} \pi_i^{Cn}(x,\theta)dF(\theta) - K(x_1)\end{aligned} \tag{16}$$

For $x_i > x_i^C$, the profit of firm i is the profit of the highest capacity firm (firm n according to our convention), given all other firm have invested the same, i. e. $x_1 = \cdots = x_{n-1}$. We get

$$\begin{aligned}\pi_n(x_n, x_{-n}^C) \quad &= \quad \int_{-\infty}^{\theta^{Cn-1}(x)} \pi_n^{C0}(x,\theta)dF(\theta) + \int_{\theta^{Cn-1}(x)}^{\theta^{Cn}(x)} \pi_n^{Cn-1}(x,\theta)dF(\theta) \\ &\quad + \int_{\theta^{Cn}(x)}^{\infty} \pi_n^{Cn}(x,\theta)dF(\theta) - K(x_1)\end{aligned} \tag{17}$$

(i) The shape of $\pi_i(x_i, x_{-i}^C)$ for $x_i > x_i^C$: The second derivative of the profit function π_n is given by[41]

$$\frac{d^2\pi_n}{(dx_n)^2} = -\frac{d\theta^{Cn}(x)}{dx_n} \underbrace{\left[\frac{d\pi_n^{Cn}(x,\theta^{Cn})}{dx_n}\right]}_{=0 \text{ (x_n is opt. at} \theta^{Cn})} f(\theta^{Cn}) + \int_{\theta^{Cn}(x)}^{\infty} \underbrace{\frac{d^2\pi_n^{Cn}(x,\theta)}{(dx_n)^2}}_{<0 \text{ by A1 part (iv)}} f(\theta)d\theta < 0. \tag{18}$$

Note that the first term cancels out and the second term is negative by concavity of the spot market profit function (implied by assumption 1). We find that for $x_i \geq x_i^C$, $\pi_i(x_i, x_{-i}^C)$ is concave, which implies that upwards deviations are not profitable.

(ii) The shape of $\pi_i(x_i, x_{-i}^C)$ for $x_i < x_i^C$: This region is more difficult to analyze since the profit function $\pi_1(x_1, x_{-1}^C)$ is not concave. We can, however, show quasiconcavity of $\pi_1(x_1, x_{-1}^C)$. For this purpose we need property 2 in order to complete the proof of existence (part I). We can show quasiconcavity of $\pi_1(x_1, x_{-1}^C)$ by showing that

$$\frac{d\pi_1(x_1^0, x_{-1}^C)}{dx_1} > \frac{d\pi_1(x_1^C, x_{-1}^C)}{dx_1} = 0 \qquad \text{for all} \quad x_1^0 < x_1^C.$$

[41] It is obvious that there is no incentive for any firm to deviate such that it is unconstrained at ∞. Thus, we only consider the case that all firms are constrained at ∞.

This holds true, since [compare also equation (15)]

$$
\begin{aligned}
\frac{d\pi_1(x_1^0, x_{-1}^C)}{dx_1} &= \underbrace{\int_{\theta^{C1}(x_1^0,x_{-1}^C)}^{\theta^{Cn}(x_1^0,x_{-1}^C)} \frac{d\pi_1^{C1}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta)}_{\geq 0 \quad \text{by property 2, part (i)}} + \int_{\theta^{Cn}(x_1^0,x_{-1}^C)}^{\infty} \frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta) \\
&\geq \int_{\theta^{Cn}(x_1^0,x_{-1}^C)}^{\infty} \frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta) \\
&= \underbrace{\int_{\theta^{Cn}(x_1^0,x_{-1}^C)}^{\theta^{Cn}(x_{-1}^C,x_{-1}^C)} \frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta)}_{\geq 0 \quad \text{by properties 1 and 2, part (ii)}} \\
&\quad + \underbrace{\int_{\theta^{Cn}(x_1^C,x_{-1}^C)}^{\infty} \left[\frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} - \frac{d\pi_1^{Cn}(x_1^C, x_{-1}^C, \theta)}{dx_1} \right] dF(\theta)}_{>0 \quad \text{by property 2, part (ii)}} \\
&\quad + \underbrace{\int_{\theta^{Cn}(x_1^C,x_{-1}^C)}^{\infty} \frac{d\pi_1^{Cn}(x_1^C, x_{-1}^C, \theta)}{dx_1} dF(\theta)}_{= \frac{d\pi_i(x^C)}{dx_i} = 0 \text{ [recall that } \theta^{C1}(x^C) = \theta^{Cn}(x^C)]} \geq 0.
\end{aligned}
$$

To summarize, in part I (i) and (ii) we have shown that $\pi_i(x_i, x_i^C)$ is quasiconcave. We conclude that the first order condition given in theorem 1 indeed characterizes equilibrium investment in the Cournot market game.

Property 2 [Properties of Marginal Profits at Stage Two] *Suppose all firms but firm 1 have invested symmetric capacities summarized in the vector x_{-1}^0. Firm 1 has invested x_1, less than each of the other firms. We obtain:*

(i) $\frac{d\pi_1^{C1}(x_1^0, x_{-1}^0, \theta)}{dx_1} \geq 0$ *for* $\theta^{C1} \leq \theta \leq \theta^{Cn}$.

(ii) $\frac{d\pi_1^{Cn}(x_1', x_{-1}^0, \theta)}{dx_1} \geq \frac{d\pi_1^{Cn}(x_1'', x_{-1}^0, \theta)}{dx_1} \geq 0 \quad$ *for* $\quad x_1' < x_1''$, $\theta^{Cn} \leq \theta \leq \infty$.

Proof (i) The first part holds due to the fact in case firm 1 is constrained, i. e. ($\theta \geq \theta^{C1}$), firm 1 would like to produce more than x_1 for all demand realizations $\theta \geq \theta^{C1}$, which, however, is not possible due to the capacity constraint.
(ii) The first inequality follows from concavity of the profit functions in the spot markets, which is implied by assumption 1. Thus, the first order condition at each spot-market is decreasing in x_1 until $\tilde{q}_i^{C0}$, which immediately yields the first inequality of part (ii). The second inequality is due to the fact that in case all firms are constrained, i. e. ($\theta \in [\theta^{Cn}, \infty]$), firm 1 would like to produce more for all demand realizations θ (which is not possible because it is constrained). $\square$

Part II: Uniqueness In this part we show that (i) x^C is the unique symmetric equilibrium and (ii) that there are no asymmetric equilibria.

(i) x^C is the unique symmetric equilibrium. If capacities are equal, i. e. $x_1^0 = x_2^0 = \cdots = x_n^0$, we have

$$\frac{d\pi_i(x^0)}{dx_i} = \int_{\theta^{Cn}(x^0)}^{\infty} [P(nx_i^0,\theta) + P_q(nx_i^0,\theta)x_i^0 - C_q(x_i^0,\theta)]f(\theta)d\theta - K_x(x_i^0).$$

Differentiation yields[42]

$$\frac{d^2\pi_i(x^0)}{(dx_i)^2} = \int_{\theta^{Cn}(x^0)}^{\infty} \left[(n+1)P_q(nx_i^0,\theta) + nP_{qq}(nx_i^0,\theta)x_i^0 - C_{qq}(x_i^0,\theta)\right] dF(\theta) - K_{xx}(x_i^0) < 0,$$

which is negative due to assumption 1. Thus, since $\frac{d\pi_i(x^C)}{dx_i} = 0$ and moreover $\pi_i(x)$ is concave along the symmetry line, no other symmetric equilibrium can exist.

(ii) There cannot exist an asymmetric equilibrium. Any candidate for an asymmetric equilibrium $\hat{x}$ can be ordered such that $\hat{x}_1 \leq \hat{x}_2 \leq \cdots \leq \hat{x}_n$, where at least one inequality has to hold strictly. This implies $\hat{x}_1 < \hat{x}_n$. The profit of firm n can be obtained by setting $i = n$ in equation (14), and the first derivative is given by

$$\frac{d\pi_n}{dx_n} = \int_{\theta^{Cn}(x)}^{\infty} \frac{d\pi_n^{Cn}(x,\theta)}{dx_n} f(\theta)d\theta - K_x(x_n).$$

It is easy to show that firm n's profit function is concave by examination of the second derivative [see equation (18)]. Thus, any asymmetric equilibrium $\hat{x}$, if it exists, must satisfy $\frac{d\pi_n(\hat{x})}{dx_n} = 0$. We now show that whenever it holds that $\frac{d\pi_n(\hat{x})}{dx_n} = 0$, firm 1's profit is increasing in x_1 at $\hat{x}$ (which implies that no asymmetric equilibria exist).

From equation (15) it follows that the first derivative of firm 1's profit function is given by

$$\frac{d\pi_1}{dx_1} = \int_{\theta^{C1}(x)}^{\theta^{C2}(x)} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta)d\theta + \cdots + \int_{\theta^{Cn}(x)}^{\infty} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta)d\theta - K_x(x_1).$$

Note that all the integrals in $\frac{d\pi_1}{dx_1}$ are positive since firm 1 is constrained at all demand realizations and therefore would want to increase its production. Thus, we have

$$\frac{d\pi_1}{dx_1} > \int_{\theta^{Cn}(x)}^{\infty} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta)d\theta - K_x(x_1),$$

where the RHS are simply the last two terms of $\frac{d\pi_1}{dx_1}$. Note furthermore that $\hat{x}_1 < \hat{x}_n$ also implies that $K_x(\hat{x}_1) < K_x(\hat{x}_n)$ (due to assumption 3) and

$$\frac{d\pi_1(\hat{x})}{dx_1} = P(\hat{x},\theta) + P_q(\hat{x},\theta)\hat{x}_1 - C_q(\hat{x}_1,\theta) < P(\hat{x},\theta) + P_q(\hat{x},\theta)\hat{x}_n - C_q(\hat{x}_n,\theta) = \frac{d\pi_n(\hat{x})}{dx_n}$$

[42] Differentiation works as in (18).

(due to assumption 1). Now we can conclude that

$$\frac{d\pi_1}{dx_1} > \int_{\theta^{Cn}(x)}^{\infty} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta)d\theta - K_x(x_1) > \int_{\theta^{Cn}(x)}^{\infty} \frac{d\pi_n^{Cn}(x,\theta)}{dx_n} f(\theta)d\theta - K_x(x_n) = 0.$$

The last equality is due to the fact that this part is equivalent to the first order condition of firm n, which is satisfied at $\hat{x}$ by construction. To summarize, we have shown that $\frac{d\pi_1}{dx_1} > 0$, which implies that there exist no asymmetric equilibria, since at any equilibrium candidate, firm 1 has an incentive to increase its capacity.

B.2 Proof of Theorem 1, Case (MC)

If the competitive outcome is implemented at stage two, firm i's stage two–profit in scenario θ is given by (12). The stage one expected profit of firm i is obtained by integrating over all profits associated with each demand realization,[43]

$$\pi_i(x, q^{FB}) = \sum_{m=0}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \pi_i^{FBm}(x,\theta)dF(\theta) - K(x_i)\,. \tag{19}$$

Thus, the first order condition is

$$\frac{d\pi_i\left(x, q^{FB}\right)}{dx_i} = \sum_{m=i}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \frac{d\pi_i^{FBm}(x,\theta)}{dx_i} dF(\theta) - K_x(x_i)\,. \tag{20}$$

Now note that $\frac{d\pi_i}{dx_i} > 0$ at $X = 0$ (since investment is gainful), that $\frac{d\pi_i}{dx_i} < 0$ for some finite value of X, and that $\frac{d\pi_i}{dx_i}$ is continuous. Thus, a corner solution is not possible, and we have at least one point where (2) is satisfied and $\frac{d\pi_i}{dx_i}$ is decreasing. Note, however, that this does not assure existence. In fact, in the scenario considered here a firm's stage one profit is not even quasiconcave, and it is not possible to reformulate the game as a supermodular game.

Now assume constant marginal cost. Note that in the case of constant marginal costs it is, independently of the capacity choices firms made at stage one, always true that either all firms are constrained at $p = C_q(\cdot,\theta)$, or none of them. Thus, it holds that $\theta^{FB1}(x) = \cdots = \theta^{FBn}(x)$.

In order to prove part (MC) of theorem 1, we apply theorem 2.1 of Amir and Lambson (2000), p. 239. They show that the standard Cournot oligopoly game has at least one symmetric equilibrium and no asymmetric equilibria whenever demand $P(\cdot)$ is continuously differentiable and decreasing, cost $C(\cdot)$ is twice continuously differentiable and nondecreasing and, moreover, the cross partial derivative $\frac{d\pi(X,q)}{dX_{-i}dX} > 0$, where X denotes total capacity

[43] We define $\theta^{FB0} = -\infty$ and $\theta^{FBn+1} = \infty$.

and X_{-i} capacity chosen by the firms other than i. In order to see that the results of Amir and Lambson apply to our setup, note that our game is equivalent to a game where firms choose output given the expected demand and cost function. Note that if the first best outcome occurs whenever capacity is sufficient, it follows that expected inverse demand is given by

$$EP(X) = \int_{-\infty}^{\theta^{FBn}(x)} P\left(Q^{FB0}\left(\theta\right), \theta\right) dF\left(\theta\right) + \int_{\theta^{FBn}(x)}^{\infty} P\left(X, \theta\right) dF\left(\theta\right), \tag{21}$$

and expected cost is given by

$$EC(x_i) = \int_{-\infty}^{\theta^{FBn}(x)} C\left(q_i^{FB0}, \theta\right) dF\left(\theta\right) + \int_{\theta^{FBn}(x)}^{\infty} C\left(x_i, \theta\right) dF\left(\theta\right) + K\left(x_i\right), \tag{22}$$

Note that $EP(X)$ is strictly decreasing in X and $EC(x_i)$ is strictly increasing in x_i, but they do not satisfy assumption 1, part (i), which is why existence and uniqueness are not implied by standard (textbook) analysis.[44] However, Amir and Lambson's assumptions[45] are satisfied, since the cross partial derivative

$$\begin{aligned} \frac{d\pi^2(X, q^C)}{dX_{-i}dX} &= -\frac{d\theta^{FBn}(x)}{dX} \underbrace{\left[-P(X, \theta^{FBn}(x)) + C_q(X - X_{-i}, \theta^{FBn}(x))\right]}_{=0 \text{ at } \theta^{FBn}(x)} f(\theta^{FBn}(x)) \\ &\quad + \int_{\theta^{FBn}(X)}^{\infty} \underbrace{\left[-P_q(X, \theta) + C_{qq}(X - X_{-i}, \theta)\right]}_{>0} f(\theta) d\theta \end{aligned}$$

is positive. This guarantees that we have at least one symmetric equilibrium and no asymmetric equilibria in case of constant marginal cost.

C Sketch of the Proof of Theorem 2

The proof of theorem 2, when welfare maximizing capacities are chosen is quite similar to the proof of theorem 1. We therefore give only a brief sketch, and refer to a working paper version of the paper (Grimm and Zoettl (2006)) for an extensive version of the proof.

[44]In fact, the expected profit function is not even quasiconcave, as it is easily seen by inspecting its second derivative. Those observations point to an error in the article of Reynolds and Wilson (2000). They make almost the same assumptions on demand as we do, but are more restrictive regarding cost (i. e. $C_q(x_i) = 0$ and $K(x_i) = kx_i$). They state (p.126 of the article) that $E[x_i P(x_i + x_{-i}, \theta) - kx_i]$ (in our notation) is strictly concave and differentiable in x_i and therefore has a unique solution. Since $E[x_i P(x_i + x_{-i}, \theta) - kx_i]$ is exactly the profit given by equation (19) for $C_q(x_i) = 0$ and $K(x_i) = kx_i$, our analysis shows that this is not true.

[45]The assumptions are: $P(\cdot)$ is continuously differentiable with $P_q(\cdot) < 0$, $C(\cdot)$ is twice continuously differentiable and nondecreasing, and $P_q(X) - C_{qq}(x_i) < 0$.

In order to prove part (FB), we consider for each realization of θ the welfare maximum at the spot market for fixed capacity choices. Integration over all realizations of uncertainty then yields expected welfare, which is given by the following expression:

$$\mathcal{W}(x, q^{FB}) = \sum_{m=0}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} W^{FBm}(x,\theta) dF(\theta) - \sum_{i=1}^{n} K(x_i). \tag{23}$$

Note that at each critical value θ^{FBm}, $m = 1, \ldots, n$, it holds that $W^{FBm-1}(x, \theta^{FBm}) = W^{FBm}(x, \theta^{FBm})$. Thus, $\mathcal{W}(x)$ is continuous. Differentiating $\mathcal{W}(x)$ yields the following first order condition:

$$\frac{d\mathcal{W}(x, q^{FB})}{dx_i} = \sum_{m=i}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \frac{dW^{FBm}(x,\theta)}{dx_i} dF(\theta) - K_x(x_i) = 0. \tag{24}$$

After verification of the second order conditions we can conclude that the above first order condition (24) yields a unique and symmetric first best solution as given stated by theorem 2, part (FB).

In order to proof part (SB), we need to determine welfare generated at the spot market at each realization of θ for fixed capacity choices given Cournot competition. Expected welfare is then again determined by integrating over all realizations of uncertainty and evaluation of first and second order conditions yields a unique and symmetric solution stated in the theorem.

D Proof of Theorem 3

Part (i) Consider the first order conditions that implicitly define total capacities in the four scenarios considered, as given in theorems 1 and 2. Recall that (i) $P_q(X, \theta) < 0$, and note that (ii) $\theta^{Cn}(x) > \theta^{FBn}(x)$ for all x. Furthermore, (iii) at (below, above) the demand realization $\theta^{Cn}(x^C)$ we have that $P_q(X^C, \theta)\frac{X^C}{n} + P(X^C, \theta) - C_q(\frac{1}{n}X^C, \theta) = 0$ (< 0, > 0). Thus, the lefthand–sides of the first order conditions can be ordered as follows:

$$\begin{aligned}
FB: &\quad \int_{\theta^{FBn}(x)}^{\infty} \left[P(X,\theta) - C_q\left(\frac{1}{n}X, \theta\right)\right] dF(\theta) \\
SB: &\quad \geq \int_{\theta^{Cn}(x)}^{\infty} \left[P(X,\theta) - C_q\left(\frac{1}{n}X, \theta\right)\right] dF(\theta) \\
C: &\quad > \int_{\theta^{Cn}(x)}^{\infty} \left[P_q(X,\theta)\frac{1}{n}X + P(X,\theta) - C_q\left(\frac{1}{n}X, \theta\right)\right] dF(\theta) \\
MC: &\quad \geq \int_{\theta^{FBn}(x)}^{\infty} \left[P_q(X,\theta)\frac{1}{n}X + P(X,\theta) - C_q\left(\frac{1}{n}X, \theta\right)\right] dF(\theta)
\end{aligned} \tag{25}$$

Note that according to theorems 1 and 2, the total capacities are determined as the values of X where the respective term equals $K_x\left(\frac{1}{n}X^Z\right)$, $Z \in \{FB, SB, C, MC\}$. Recall that in all cases we get interior solutions and note that the above terms (except for the one that determines X^{MC}) are decreasing in X, while K_x is increasing in X. This immediately implies $X^{FB} \geq X^{SB} > X^C$.

In order to see why the ranking stated in the theorem also holds for case MC, note that the above term in scenario C is strictly decreasing in X, whereas in scenario MC it satisfies $LHS(0) > K_x(0)$ (since production is gainful) and $LHS(X) < K_x(X)$ for X high enough. Since $K_x(X)$ is increasing in X, this immediately implies that for any equilibrium investment X^{MC} it holds that $X^C \geq X^{MC}$.

Part (ii) As n approaches infinity, all first order conditions collapse to $\int_{-\infty}^{\infty}[P(X,\theta) - C_q(0,\theta)]dF(\theta) = K_x(0)$.

E Proof of Theorem 4

Let x^0 be a vector of equal capacities summing up to X^0. We have $\theta^{FBn}(x^0) \leq \theta^{Cn}(x^0)$ for all x^0 and both, $\theta^{FBn}(x^0)$ and $\theta^{Cn}(x^0)$ are increasing in X^0.

(i) If $X^C \leq Q^C(\underline{\theta})$, then $\theta^{Cn}(x^C) \leq \underline{\theta}$, since installed capacities are lower than the unconstrained Cournot-output at the lowest realization of uncertainty with positive weight denoted by $\underline{\theta}$. This implies that $\theta^{FBn}(x^{MC}) \leq \theta^{Cn}(x^C) \leq \underline{\theta}$, since $X^{MC} \leq X^C$.

Then the first order conditions for the cases (C) and (MC) in theorem 1 collapse since $f(\theta) = 0$ for all $\theta \in [\theta^{FBn}(x^{MC}); \underline{\theta}]$ and thus, the lower limit of integration is given by $\underline{\theta}$. This proves "$\Rightarrow$". In order to prove "$\Leftarrow$", note that $X^C > Q^C(\underline{\theta})$ implies $\underline{\theta} \leq \theta^{FBn}(x^{MC}) < \theta^{Cn}(x^C)$.Then the the lower limit of integration in first order conditions for the cases (C) and (MC) does not coincide which implies $X^{MC} < X^C$ if $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

(ii) The proof works analogously to part (i).